BUSINESS SCHOOL

AMERICAN

GEORGIAN AMERICAN
UNIVERSITY

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"# $#!%&'( ) (*+&, -+'+!.! /) 012+) 345!67) 01'&!+4!89: (4+*(5+14('!%45, 91; 1'1: 7!

<# 6#!=3>3'+2*3!.! ?@) (4!A3&1@9B3! C (4(: 3) 345!D, +'1&1; , 7!+4!E12(7F&! 89: (4+*(5+14!

G# E#!H(0@4(&, -+'+!.!A1'3!1I!J3(239&, +; !D15345+('!I19! C (4(: 39&!

K# H#!L, +>-+4+2*3!.!A3) (9>!14!A+: , 5!L145+4@1@&! / M; 14345+('! C (95+4: ('3&!

N# ?#!=+: @9(2*3!.!A1'3!1I! ?@) (4!L(; +5(4!04!L1453) ; 19(97! 89: (4+*(5+14&!

P# Q#!=('(42(9+&, -+'+R!E#!E1914S(2*3R!E#!T*@4(&, -+'+!.! C123'+4: !) 32+B('!; 91B32@93!; 9151B1'&! (21; 5+14!; 9+19+5+3&!+4!, 3('5, B(93!

U# =#!=+: @9(2*3.!H@+'2+4: ! ?+: , !D39I19) (4B3! V 19>; '(B3!+4!W319: +(4! 89: (4+*(5+14&!

X# E#!=@5('+R!A#!E3-*(2*3!.!T5+'+57!) (M+) +*(5+14!; 910'3) !Y+5, !@4B395(+457!1I!&@BB3&&!; 910(0+'+57!

Z# C#! C (4+(R!A#!E3-*(2*3!.!%! C (95+4: ('3!L, (9(B539+*(5+14!1I!5, 3!W3439('!61'@5+14!1I! [@(29(5+B! \@4B5+14('! /] @(5+14!

"^# _#! ` abcde ` f ` !.!g ` h i j f ` !kc i _cImn ` o!op i cpqk ` qb ` !mg ` i q! _c!ocd j c f a!b ` r nqod ` !! ocIc i seq f ao!mckc f ` s r q!

""# J#!W(B, 3B, +'(2*3R!E#!T*@4(&, -+'+R!E#!E1914S(2*3!.!04B19; 19(5+4: ! C (B, +43!J3(94+4: ! C123'&!+4! t ('@(5+14!1I! ?32: +4: !659(53: +3&!

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H I JKLM I H@D!NO I JKPN!M@!KQ67@MR7DMK@7P!7@DSQKTKPK6O!

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"-./-0!#$!1-.#%(!2+#(-'-3+45!20-67''-05!!

1.#7$/#6#.!80-93!:7);705!<9'#$7''!1.+--(5!!

87-0=#%$!&)70#.%$!>$#,70'#/45!!!

?@A?5!87-0=#%5!B;#(#'#5!&(7C'#D*7!1/077/!@?!

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"#$%&"'%!

B+7!3%370!307'7$/'!'-)7!076(7./#-$'!60-)!/+7!-0=%$#*%/#-$%(!%$/+0-3-(-=4!,#7E!-$!/+7!30-.7''7'!#$!-0=%$#*%/#-$'! $-E%D%4'5!)77/#$=!/+7!.+%((7$=7'!-6!07)-/7!E-0C5!'+#6/!-6!)%$%=7)7$/!30-.7''!60-)!/+7!/0%D#/#-$%(!-66#.7!/-!/+7! %;'/0%./!,#0/9%(!%07%F! B+7!3%370!307'7$/'!/+7!07'9(/'!-6!/+7!'%)3(7!07'7%0.+5!#$/70307/#$=!/+7)!#$!/+7!6#,7! -0=%$#*%/#-$%(!'4);-('!60%)7E-0CF! B+707!%07!)%D7!'-)7!.-$.(9'#-$'!-$!/+7!'4);-('!7);-D#)7$/!#$! -0=%$#*%/#-$%(!.9(/907!%$D!/+7#0!#)3%./!-$!/+7!#$,-(,7)7$/!$-/!%'!%$!//#/9D7!;9/!%'!%!,%(97!#$!$-E%D%4'!7.-$-)4! -6!3%0/#.#3%/#-$F!!!!

G74!E-0D'H!-0=%$#*%/#-$%(!%$/+0-3-(-=45!-0=%$#%/#-$%(!'4);-('5!-0=%$#*%/#-$%(!.9(/907!*!

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>6/)-056)/!.4)!5 : @9-.,6+)!98!89+>*56?!96!.4)!2850.)-*;!56!4> : ,6!-),*9656?1!,*!Q&),0!4> : ,6! D)56?*!,-)!69.1!89-!.4)! : 9*.!@,-.1!56!+96*+59>*!+96.-90!98C9-!)B)6!+96*+59>*0E!,<,-)!98C.4)5-! -),*9656?A! L 9*.!98!.4)5-!-),*961!D)*5/)*1!5*!D,*)/!96!B,-59>*!756/*!98!@-9.9.E@)*1!8-, : 56?*1!,6/! :).,@49-*R! FP,79881! STTT1! @A! SUGA! H4)69 :)6909?E! V>)*.596)/!.4)!6,.>-)!98!+9?65.5961!,6/! *.,.)/!.4)!@-56+5@0)!23!, : !.4)-)89-)!3!8))0!49<!3!, : !.456756?!96!*9 :).456?;A!%45*!7)E! : 9 :)6.! 98!856/56?!9>.!.4)!8))056?!49<!96)!.4567*!,D9>.!*9 :).456?1! : ,7)*!9-?,65K,.596,0!,6.4-9@909?E! *.>/5)*!.9! : 9B)!8-9 : !.4)!@-56+5@0)!2<4,.!5*;!.9!.4)!@,-,/5? : !249<!5.!5*;A!H-9D,D0E1!.4)!960E! +966)+.596!<5.4!.45*!,*@)+.!98!)+969 : 5+!-)0,.596*45@*!56!9-?,65K,.596*!+,6!D)!*).!.49>?4!.4)! *E : D90*1!*@)+585+,00E1!96)!98!.4)!*E : D90;*!/5*@0,E1!C! :).,@49-A!!

W-?,65K,.596!5*!,! : 565C*, : @0)!98!.4)!+>0.>-)!,6/!*9+5).E1!98.)6!5*!/5*@0,E)/!,*!,!@-9.9.E@)A! J9<,/,E*!4> : ,6756/!5*! : 9B56?!.4-9>?4!.4)!.-,6*89- : ,.596!98!*9 :)!B,0>)*1! : ,6E!@-9+)**)*! ,-)!56.)-B)6)/1!,6/!.4)!90/!,@@-9,+4)*!8,50!.9!4)0@!9-?,65K,.596*!9@)-,.)1!@)-89- : !,6/!?).! -)*>0.*A!%4)-)!5*!,!6))/!98!*9 :)!56.)?-,.)/!,@@-9,+4!56! : ,6,?56?!.4)!*>DM)+.5B)!+96.)N.!98!) : D9/5 :)6.A!%4)!)+969 : E!+4,6?)*!8-9 : !.4)!)+969 : E!98!9D*)-B56?!.9<,-/*!.4)!)+969 : E!98! 56B90B) :)6.A!H)9@0)!<,6.!.9!/9! : ,6E!.456?*!.4) : *)0B)*1!.4)E!<,6.!.9! : ,7)!@5+.>-)*1!/-,<1! <-5.)!*.9-5)*1!D)!5680>)6+)-*!,6/! : 9.5B,.596,0!*@),7)-*1!/9!.4)5-!9<6!D09?*1!96056)!*+4990*!,6/! D>*56)**)*A! X+969 : E!98!,+.5B)!@,-.5+5@,.596!5*!D,*)/!96!.4)!5/),!98!) : D9/5 :)6.1!C!D>50/56?! ,<,-)6)**!,6/! : 56/8>06)**A!X : @09E))*!<5.4!,!?-),.)-!,.)6.596!<,6.!.9!856/!.4)!*)6*)!56!<4,.! .4)E!/9!,6/!*))! : 9-)!9@.596*!89-!.4)!+495+)A!%45*!,+.5B)!56B90B) :)6.!56!)B)-E.456?!.4)E!/91! : ,7)!9-?,65K,.596*!.9!-) : 9.)!69.!960E!*9 :)!<9-71!D)+,>*)!98!.4)!@,6/) : 5+1!D>.!,*!<)00!*9 :)! : ,6,?)-5,0!8>6+.596*A!%4)!,D*)6+)!98!.4)!.-,/5.596,0!9885+)!,6/!*9 :)!D,*5+!-5.>,0*!F89-!)N, : @0)1! ?-)).56?!*)+>-5.E!56!.4)! : 9-656?1!9-!?956?!.9!0>6+4G! : ,/)!.4)!+4,00)6?)!98!56B90B) :)6.! : 9-)! 9DB59>*!89-!9-?,65K,.596*A!!

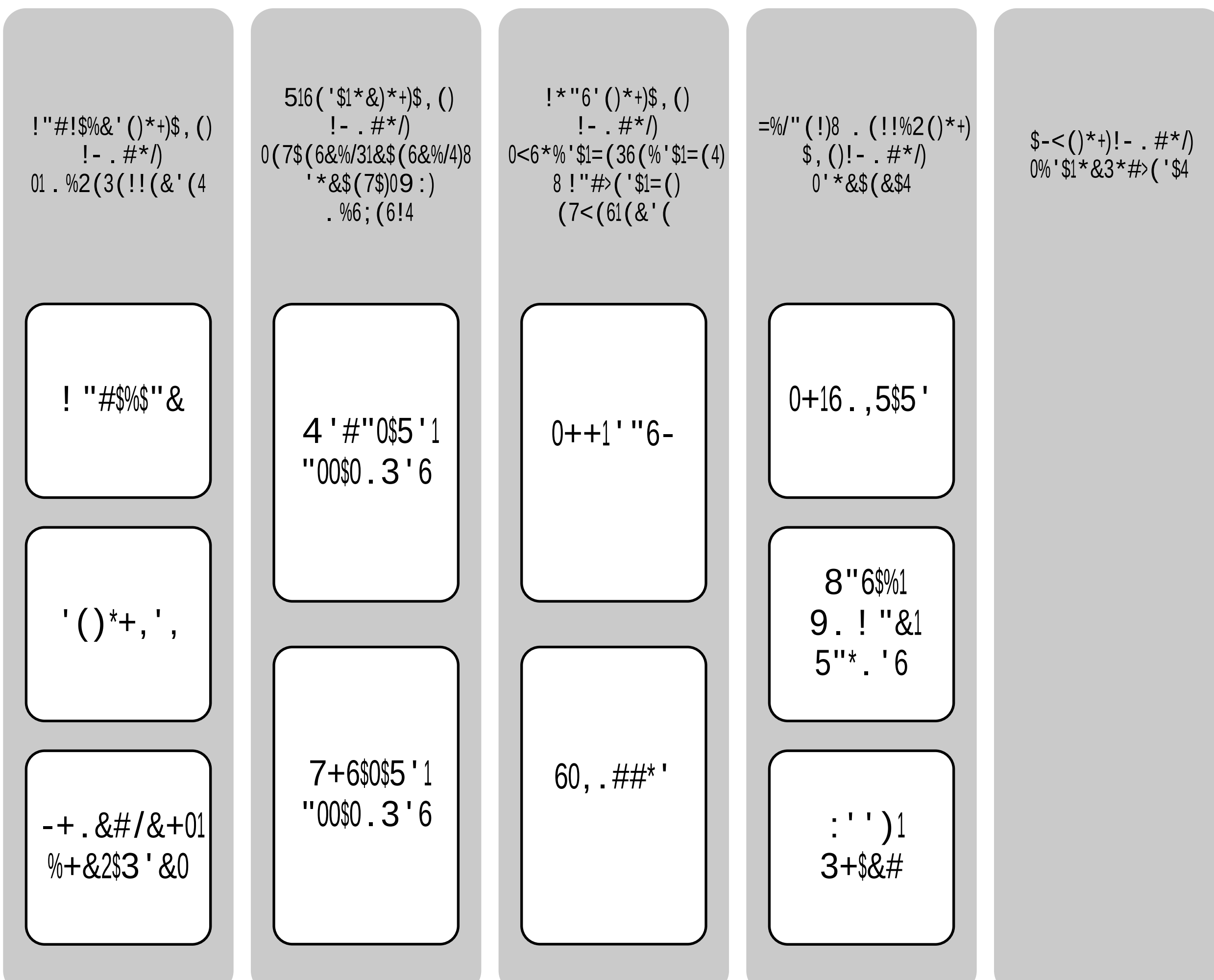
%4)-)!5*!,!*458.!56!)88)+.5B)6)**!8-9 : !,/,@.,.596!F9-5)6.,.596!@-9?-, : *G!.9!56B90B) :)6.!F,+.5B)! *4,-56?!98!*9 :)!B,0>)*!,6/! :).,@49-*GA!36!.45*!*458.! : 9-)!9DB59>*!5*!,!6))/!89-!*E : D90*!.9!D)! 56.)?-,.)/!56!.4)!9-?,65K,.596,0!*9+5,0!,6/!+>0.>-,0!@-9+)**)*A! I 5?4!0)B)0!98!56B90B) :)6.!,6/!

.-96?)-! 5/)6.5.E! <500! D)!).,D05*4)/! ,*! ,! +96*)V>)6+)! 98! .4)! @,6/) : 5+! 5680>)6+)! <)! ,-)!)N@)-5)6+56?!69<A!W-?,65K,.596,0!+>0.>-)!5*!,!*E : D905+!8-, :)!89-!) : D9/5 :)6.!98!.4)!*)6*)1! +495+)1!,<,-)6)**!98!.4)!M9D1!9-!98!<9-7!56!?)6)-,0A!!!!

56.)-@-).,.596!98!.4)!) : @09E :)6.!)N@)-5)6+)!8-9 : !.4)!-)*@96/)6.*A!%4)!-)*@96/)6.*!<)-)! 988)-)/!.9!<-5.)!,!8,5-EC.,0)!98!.4)5-!<9-7!)N@)-5)6+)1!89009<56?!*9 :)!+9,+456?!V>)*.596*A!`a! -)*@96/)6.*!F8-9 : !O)9-?5,61!&>**5,6!,6/!56.)-6,.596,0!D>*56)**!9-?,65K,.596*G1!/)*+-5D56?! .4)5-!<9-7!) : D9/5 :)6.!)N@)-5)6+)1!89-!.4)! : ,56!+4,-,+.)-!F4)-9G! : 9*.0E!,@@),0!.9!.4-))!D,*5+! .E@)*^! : ,?5+5,61!)N@09-)-!9-!*9 :)96)!E9>6?!F69.!)N@)-5)6+)/GA!!

3.!>6/)-056)*!.4)!5 : @9-.,6+)!98!56B90B) :)6.!F9-!)B)6!5/)6.585+,.596!<5.4!.4)!<9-7G1!,*! :)6.596)/!,D9B)A!]9009<56?!.4)!85B)!]>00)-;*!/5 :)6*596*1!<)!+,6!D>50/!.4)!.,D0)!98! :).,C :).,@49-*!8-9 : !.4)!*>-B)E!F$))!XN45D5.!SGA!

XN45D5.!S^!



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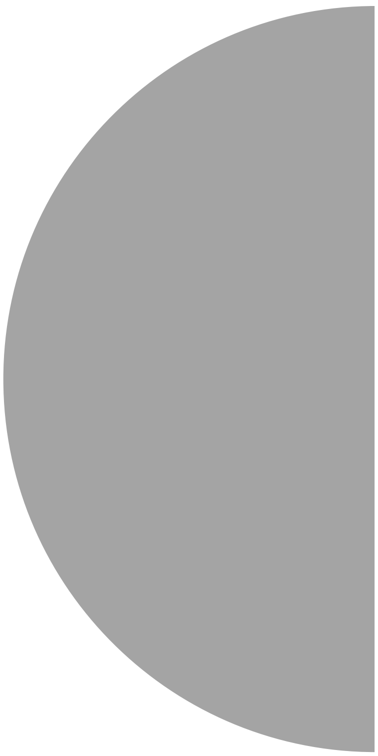
%4)!D)4,B59-!@,..)-6*!,-)!89-:)/!DE!*)B)-,0!0,E)-*1!057)!4>:,6!+9-)!F*9:).456?!<)!,00!4,B)! 56!+9::96!8-9:!49:9!*,@5)6*!*,@5)6*G1!.4)!56.)-?)6)-,.596,0!@,..)-6*!<)!564)-5.1!*9:)! @)-*96,0!.-,5.!*56!+4,-,+.)-A!%4)*)!@,..)-6*!,-)!/5*@0,E)/!56!-)0,.596,01!<9-71!+>0.>-,0!+96.)N.! 56!.4)!89009<56?!+9--)0,.596!56!0,E)-*!F*)!XN45D5.!ZGA!!!

!!XN45D5.!Z^!



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W-?,65K,.596,0!+>0.>-)!5*!,6!56.)?-,.)/!85)0/!.4,.!56+0>/)!,00!.4)*)!)0):)6.*1!5.!5*!69.!M>*.!.4)! +>0.>-,0!+96.)N.1!5.!,6!56.)?-,.)/!2/9M9;!89-!D)4,B59-,0!,6/!+9::>65+,.5B)!@,..)-6*1!,6/!.4)! V>,05.E1!/)@.4!,6/!56.)6*5.E!98!):D9/5:)6.!5*!/)856)/!DE!.4)!*E:D90*;!*.-)6?.4A!W-?,65K,.596,0! +>0.>-)!5*!D,*)/!96!.4)!*E:D905+!:),656?*!/)85656?!4>:,6756/1!057)!@-5:,-E!*)6*)1!,*!b>B,0! I,-,-5!*.,.)*!56!45*!D997!Q$,@5)6*R!F I,-,-51!Z[SaG1!I9:9!$,@5)6*!5*!.-,6*+)6/)/!.4)!D5909?5+,0! *>-B5B,0!05:5.*1!D-),756?!.4)!0,<*!98!6,.>-,0!*)0)+.5961!,6/!*.,-.56?!.4)!0,<*!98!56.)005?)6.!

/)*5?6*A!H)9@0)!5*!,6!5 : @9-.,6.!)0) :)6.!56!>6/)-*.,6/56?!9-?,65K,.5961!,096?!<5.4!*.->+.>-)! ,6/!@>-@9*)1!*9!4> : ,6!+,@5.,0!5*!.4)! : 9*.!/E6, : 5+!,6/!56B,0>,D0)!-)*9>-+)!98!.4)!9-?,65K,.596A! L9-)9B)-1!56B90B) :)6.1!)6?,?) :)6.1!@,-.5+5@,.596!,-)!,0-),/E!69.!.4)!,..5.>/)*1!D>.!5 : @9-.,6.! B,0>)*!.4,.! : ,..)-!56!9-?,65K,.596,0!*>-B5B,0!,6/!6)<! :).,@49-*!89- : >0,.596A!!

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&)8)-)6+)*^!

"0B)**961! LA!FZ[[ZGA! \6/)-*.,6/56?!W-?,65K,.596,0! '>0.>-)1!P9*! "6?)0)*^!$"OXA!

]>00)-1!$A&A!FZ[SSGA!W-?,65K,.596,0!$E : D905* : ^! "! L>0.5C

S8 I 7@!QHNK8Q9H! I 7@76H I H@D!TSMPKNKTSO!M@!DKL7OUN!

KQ67@MR7DMK@!

1%(-) 7!G7C7(#D*7!

2+"!20-=0%)!'/9D7$/5!!

87-0=#%$!&)70#.%$!>$#,70'#/45!!!

?@A?5!87-0=#%5!B;#(#'#5!&(7C'#D*7!1/077/!@?!

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"#$%&"'%!

B+7!)%#$!7''7$.7!-6!/+7!/-3#.!K!#'!+9)%$!07'-90.7'5!E+#.+!076(7./'!/+7!E7%(/+!-6!%$4!'-.#7/4!%$D!6-0!/+7! D7,7(-3)7$/!#/!#'!3-''#;(7!/-!30-D9.75!#)30-,7!%$D!9/#(#*7!/+7!'%)7!07'-90.7'5!/%C#$=!#$/-!.-$'#D70%/#-$!/+7! #$/707'/'!-6!7%.+!#$D#,#D9%(F!L9)%$!,#7E'!%07!;%'7D!-$!+#'!C$-E(7D=75!J9%(#6#.%/#-$'!%$D!7M370#7$.7F!B+#'!%('-! 07670'!/-!/+7!)%$%=7)7$/!-6!+9)%$!07'-90.7'F!&$4!D7.#'#-$!07=%0D#$=!/+7!370'-$$7(5!E+#.+!#'!/%C7$!;4!/+7!

@)-89- : ,6+)1!V>,05.E1!-)+9?65.596!98!)6B5-96 :)6.*!,6/!-5?4.*!56!) : @09E :)6.!-)0,.596*A!3.!5*!
: 9-)!*.-,.)?5+!,6/!096?!.)- : A!!

%4)!+-5.5+,0!@)-*@)+.5B)!98! I&L!5*!,6!9>.+9 :)!98!69- : ,.5B)!@)-+)@.596A!3.!@-9@9*)*!.4,.!9-?,65K,.596*! : ,56.,56!.4)5-!f*98.!I&L f!,@@-9,+4!960E!.9!*49<!56!.4)5-!@905+5)*!D>.!56!-),05.E1!.4)E!@-,+.5+)!f4,-/!I&L f!.9!)N.)6/! : ,6,?) :)6.!+96.-90A!

%4)E!@-).)6/!.9!D)!+96+)-6)/!89-!<9-7)-*!,6/!)N@095.!.4) : !.4-9>?4!<9-7!56.)6*585+,.596!,6/!/9<6*5K56?A!%4>*1!'-5.5+,0!H)-*@)+.5B)!@-9@9*)*!.4,.!I&L!4,*!960E!+4,6?)/!9-?,65K,.596,0!-4).9-5+!,6/!-),05.E!4,*!69.!+4,6?)/!*56+)!.4)!56.-9/>+.596!98!H)-*966)0A!I9<)B)-1!5.!,0*9!,-?>)*!.4,.!I&L!>*)*!,!>65.,-E1!*98.!I&L!-4).9-5+!.9!9D*+>-)!4,-/!-),05.E!+4,-,+.)-5K)/!DE!56+-),*)/! : ,6,?) :)6.!+96.-90!,6/!/5 : 565*4)/!M9D!*)+>-5.E!89-!) : @09E))*A!%4)!85-*.!@-9@9*5.596!/)*+-5D)*!I&L!,*!@9<)-0)**!,6/!.4)!*)+96/!,*!@9<)-8>0A!!

#)4,B59-,0!@)-*@)+.5B)!98!I&L!D)05)B)*!.4,.!5.!5*!B5.,0!89-!,6!9-?,65K,.596!.9!+96.-90!.4)!D)4,B59-!98!5.*!) : @09E))*!.9!D-56?!.4)!/)*5-)/!-)*>0.*!8-9 : !.4) : A!]9+>*!5*!96!.4)!5/)6.585+,.596!98!/)*5-)/!D)4,B59-1!)6*>-56?!,B,50,D505.E!98!9@@9-.>65.5)*!,6/!)6B5-96 :)6.!89-!/)*5-)/!D)4,B59-1!/)B)09@56?!) : @09E))*e!*7500*!.9!D-56?!/)*5-)/!D)4,B59-1!,6/! : 9.5B,.56?!) : @09E))*!.9!D)4,B)!,*!/)*5-)/A!!

$.-,.)?5+!@)-*@)+.5B)!98!I&L!D)05)B)*!.4,.!.4)!4> : ,6!-)*9>-+)*!,-)!B,0>,D0)!.6505-34 : -(9)0.2(8)0.1(!)-3.3(

: ,6>8,+.>-56?!,6/!.4)!.-,/5.596,0!*)+.9-*1!.4)!6))/!.9!-) : ,56!+9 : @).5.5B)!4,*! :),6.!.4,.!85- : *! 56!.4)*)!*)+.9-*!/)@09E!*.-,.)?5)*!.4,.! : ,7)!)88)+.5B)!>*)!98!.4)5-!-)*9>-+)*A!%45*!+4,6?)/! D>*56)**!0,6/*+,@)!4,*!+9 :)!,D9>.!,*!,!-)*>0.!98!,!@,-,/5? : !*458.!56!.4)!<,E!D>*56)**)*!,6/! 85- : *!B5)<!.4)5-!) : @09E))*!,*! : 9-)!.4,6!M>*.!-)*9>-+)*!,6/!56*.),/!,/9@.!,!Q@)9@0)!85-*.R! ,@@-9,+4A!

%4)!/)B)09@ :)6.!98!.4)!I&!L,6,?) :)6.!H4509*9@4E!5*!,!096?C.)- : !@-9+)**A!%4)!@4509*9@4E! 5*!>*>,00E!5689- : ,0!,6/!-)*@)+.*!B,0>)*!,6/!9@56596*!98!.4)! : ,56!*.,7)490/)-*A!%4)!0),/)-!98! .4)!9-?,65K,.596!4,*!,!*5?6585+,6.!5680>)6+)!96!.4)!I&!H4509*9@4EA!%4)!I&!P),/)-!5*!.4)!6)N.! 96)!5680>)6+56?!.4)!@4509*9@4E!*5?6585+,6.0EA!

%4)! : 9/)-6!0),/)-!>*>,00E!-)V>5-)*!,! : 9/)-6!I&!L,6,?) :)6.!,@@-9,+4!,6/!D>50/*!.4)!)6B5-96 :)6.!*>5.,D0)!89-!.4)!)B90>.596!98!.4)! : 9/)-6!I&!L,6,?) :)6.A!%45*!5*!.4)!I&! L,6,?) :)6.!H4509*9@4E!D)456/!.4)!*+)6)A!

%4)! : ,56!8,+.9-*!5680>)6+56?!.4)!I&!@4509*9@4E!56!.4)!9-?,65K,.596!,-)^!

- P),/)-*45@!$.E0)!
- '9-@9-,.)!'>0.>-)!
- '9-@9-,.)!g,0>)*!
- L,-7).!'9 : @).5.596!

%4)!0),/)-*45@!*.E0)!5*!)N.-) :)0E!5 : @9-.,6.A!%4)!0),/)-*!98!.4)!9-?,65K,.596!,-)!.4)!-90)! : 9/)0*!89-! : ,6,?)-*!,6/!) : @09E))*A!%4)! : ,6,?)-!,0<,E*!.-5)*!.9!,+.!,*!.4)!0),/)-!/9)*A!%4)! D)4,B59-!5*!9D*)-B)/!,6/!*@-),/!,+-9**!.4)!)6.5-)!9-?,65K,.596A!%4)!0),/)-!*).*!.4)!D,*5+!)N@)+.,.596*!8-9 : !,00!) : @09E))*!,6/! : ,6,?)-*A!%4)E!,/M>*.!.4)5-!D)4,B59-!.9!D)!8>00E! +9 : @05,6.!<5.4!.4)!0),/)-;*!)N@)+.,.596*A!%4)!I&!L,6,?) :)6.!H4509*9@4E!5*!.4)!*, :)!*.9-EA!

%4)!9-?,65K,.596*!>*56?!.4)!@-9/>+.5B5.ECD,*)/!+9685?>-,.5961!@)-89- : ,6+)!9-5)6.)/!@-9+)**)*!
<)-)!>*)/!<5.456!),+4! I &!@-,+.5+)!.9! : ,N5 : 5K)!@-9/>+.5B5.E!D>.!<)-)!69.!>*)/!.9!)64,6+)!
56.)?-,.596!D).<))6! I &!@-,+.5+)*A! "*!,!-)*>0.1!),+4! I &!@-,+.5+)!9@)-,.)/! : 9-)!9-!0)**!,*!,!
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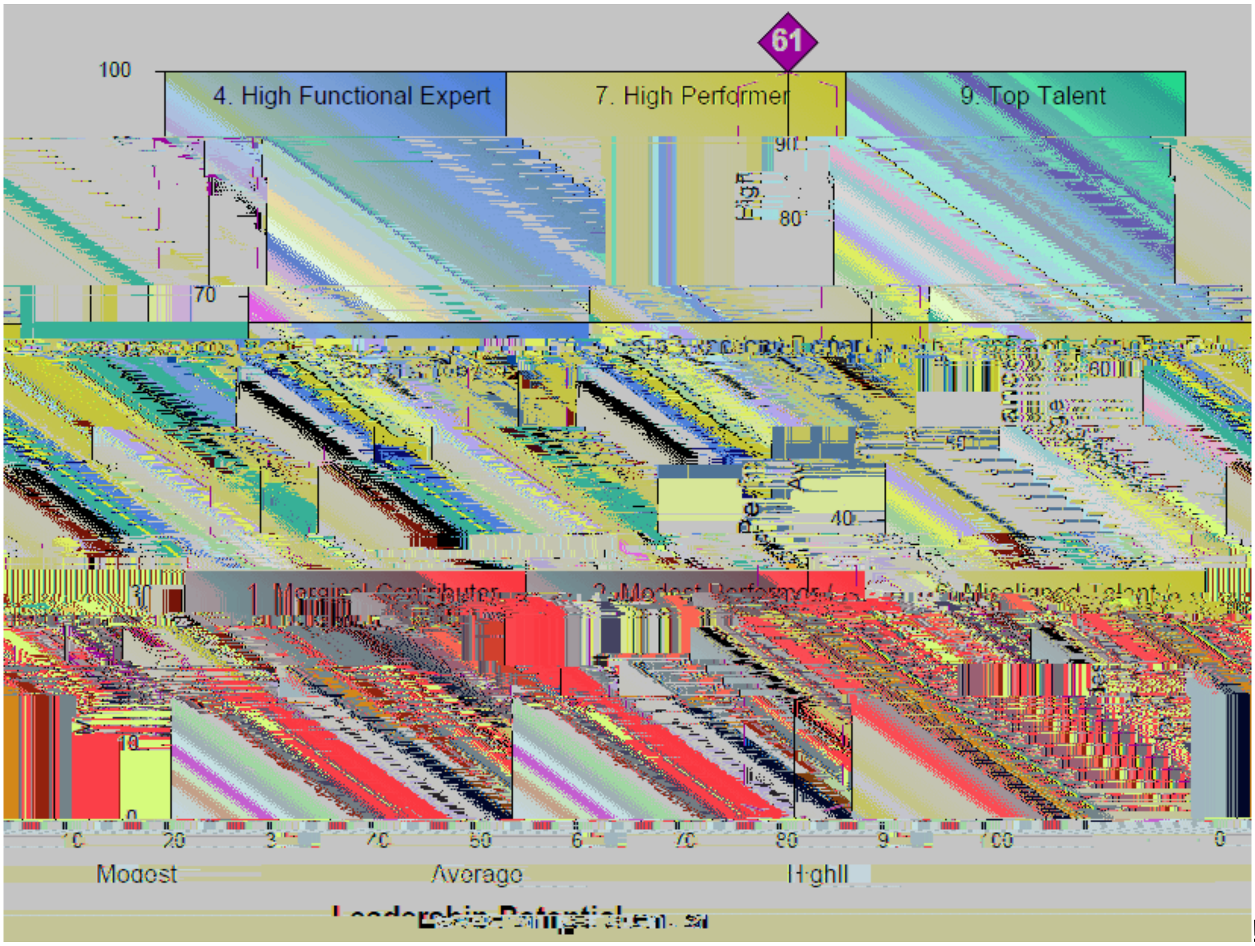

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P),/)-*45@!/)B)09@ :)6.!5*!,!7)E!8,+).!98!9-?,65K,.596,0!*>++)**A!! ( )B)09@56?!0),/)-*!,6/! 0),/)-*45@!.), : *!+,6!)64,6+)!(5)-0.2(6)0.1(?)-0.1(!<9)0.2(9 I 1831(>-63550337.5515T50337.521Tm(!)Tj6032.3(201Tf

Remark on Right Continuous Exponential Martingales

where M^c denotes a continuous martingale part of M. It is well known that ${}_t(M) = 1 + {}_0^t \; {}_{s-}(M)dM_s$, so it is clear that for local martingale M the additional increasing process B_t and the compensator, but we have restriction on jumps of M. In their result Larsson and Ruf [3] used the predictable stopping time τ and they have additional restriction on M to obtain the set equality. We also have additional but di erent kind of restriction on jumps on M and with this we use any kind of stopping times.

Proof of the Theorem 1: It is well known, that $M = M^c + M^d$ where M^c is

The supermartingale property of $\left(\frac{1}{2}M^d\right)$ implies P $_T(\frac{1}{2}M^d) <$ = 1, so we obtain that $\{$ $_{s\leq T}$ ln $\left(1 + \frac{1}{4} \cdot \frac{(\triangle M_s)^2}{1+\triangle M_s}\right)$ = $\}$ $_T(M^d)$ = 0 . Now we will have the following chain of set inclusion and equality:

$$\sum_{s\leq T} \frac{(\quad M_s)^2}{1 + \quad M_s} = \qquad = \quad 1 + \frac{1}{4} \sum_{s\leq T} \frac{(\quad M_s)^2}{1 + \quad M_s} =$$

$$\prod_{s\leq T} \left(1 + \frac{1}{4} \cdot \frac{(\quad M_s)^2}{1 + \quad M_s} \right. = \qquad = \quad e^{\sum_{s\leq T} \ln\left(1+\frac{1}{4}\cdot\frac{(\triangle M_s)^2}{1+\triangle M_s}\right)} = \qquad =$$

$$\sum_{s\leq T} \ln\left(1 + \frac{1}{4} \cdot \frac{(\quad M_s)^2}{1 + \quad M_s}\right. = \qquad {}_T(M^d) = 0 \ .$$

Now it is time to prove the reverse set inclusion: $_T(M^d)$ = 0 $_{s\leq T} \frac{(\triangle M_s)^2}{1+\triangle M_s}$ = . Let $\mu(\omega, t, x)$ be the jump counting measure for local martingale M and $\nu(\omega, t, x)$ be its compensator. Then we have the representation: $M^d_t = \int_0^t \int_{[-1;+\infty)} x d(\mu - \nu) = \sum_{s\leq t}$

Inequality $M_s^1 >$ 1 implies that $M_s^1 \prec \frac{2(\triangle M_s^1)^2}{1+\triangle M_s^1}$, so using this we obtain:

$$_T(M^1) = 0 \quad \sum_{s\le T} \frac{2(M_s^1)^2}{1+ M_s^1} = \quad = \quad \sum_{s\le T} \frac{(M_s^1)^2}{1+ M_s^1} = \quad .$$

II. Now we will show that $_T(M^2) = 0 \{ \sum_{s\le T} \frac{(\triangle M_s^2)^2}{1+\triangle M_s^2} = \}$.

$$_T(M^2) \; ^2_T\Big(-\frac{1}{2}M^2\Big) = \exp \; M_T^2 + \sum_{s\le T}\big[\ln(1+ M_s^2) - M_s^2\big] - M_T^2 +$$

$$\sum_{s\le T}\big[2\ln(1-\frac{1}{2} M_s^2) + M_s^2\big] = \exp \sum_{s\le T} \ln(1+ M_s^2)\big(1-\frac{1}{2} M_s^2\big)^2 .$$

From the last equality and the supermartingale property of $\big(-\frac{1}{2}M^d\big)$ we deduce that

$$\{ _T(M^2) = 0\} \quad -\sum_{s\le T} \ln(1+ M_s^2)\big(1-\frac{1}{2} M_s^2\big)^2 = \quad . \qquad (2)$$

Using $M_s^2 < 1$ and Lemma 1 from Appendix we obtain $-\ln(1+ M_s^2)\big(1-\frac{1}{2} M_s^2\big)^2 \prec \frac{2(\triangle M_s^2)^2}{1+\triangle M_s^2}$ and this with (1) gives us inclusion:

$$_T(M^2) = 0 \quad \sum_{s\le T} \frac{2(M_s^2)^2}{1+ M_s^2} = \quad = \quad \sum_{s\le T} \frac{(M_s^2)^2}{1+ M_s^2} = \quad .$$

Now if we summarize results of I and II, we will have:

$$_T(M^d) = 0 \; = \; _T(M^1) = 0 \quad _T(M^2) = 0$$

$$\sum_{s\le T} \frac{(M_s^1)^2}{1+ M_s^1} = \quad \sum_{s\le T} \frac{(M_s^2)^2}{1+ M_s^2} = \quad =$$

$$\sum_{s\le T} \frac{(M_s^1)^2}{1+ M_s^1} + \sum_{s\le T} \frac{(M_s^2)^2}{1+ M_s^2} = \quad = \quad \sum_{s\le T} \frac{(M_s)^2}{1+ M_s} = \quad .$$

Finally we obtain $_T(M^d) = 0 \quad = \quad \sum_{s\le T} \frac{(\triangle M_s)^2}{1+\triangle M_s} =$ and

$$_T(M) = 0 \quad = \quad _T(M^c) = 0 \qquad _T(M^d) = 0 \quad =$$

$$M^c \,_T = \sum_{s\le T} \frac{(\quad M_s)^2}{1 + \quad M_s} = \quad = \quad M^c \,_T + \sum_{s\le T} \frac{(\quad M_s)^2}{1 + \quad M_s} = \quad .$$

□

4. A .

1. $k(x) = \frac{2x^2}{1+x} + \ln(1+x)(1-\frac{1}{2}x)^2 \geq 0$ for any $x \quad (-1;1)$.

Proof.

$$k'(x) = \frac{4x(1+x) - 2x^2}{(1+x)^2} + \frac{(1-\frac{1}{2}x)^2 - (1+x)(1-\frac{1}{2}x)}{(1+x)(1-\frac{1}{2}x)^2} =$$

$$\frac{2x^2+4x}{(1+x)^2} - \frac{3x}{(1+x)(2-x)} = \frac{-2x^3-3x^2+5x}{(1+x)^2(2-x)} = \frac{x(2x+5)(1-x)}{(1+x)^2(2-x)}.$$

It is obvious that $k'(0) = 0$, $k'(x) < 0$ when $x \quad (-1;0)$ and $k'(x) > 0$ when $x \quad (0;1)$. So $x = 0$ is a minimum point and because $k(0) = 0$, we can deduce that $k(x) \geq 0$ for $x \quad (-1;1)$. □

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)*-!%-&1!4;!G*91'0!(',+%'/X!+$!'!:-8!-/-1-0%!%4!+1,&46-!%*-!'$$-%$!4;!%*-!4&2'0+C'%+40<!D%!('0!#-!$--0!'$!9%'+0'#/-!(41,-%+%+6-!'56'0%'2-!'05!'!;'(%4&!4;!+0(&-'$-5!-1,/48--!-;;+(+-0(8<!K41-!%*-4&+$%$!',,/8!&9/-$!%4!,&46-!%*-!'#+/+%8!%4!(&-'%-!9$-;9/!(41,-%+%+40$!#-%7--0!(41,'0+-$!#8!5-6-/4,+02!+05+6+59'/!*91'0!&-$49&(-$<!

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Building High Performance Workplace in Georgian Organizations

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ABSTRACT

Many organizations require collaboration to achieve projects or organizational goals. Teamwork can help solve difficult problems and improve productivity. High-performance teams in the workplace make this success possible by combining individual talent to achieve a common goal. Since the 1980s, organizations and companies placed an increasing number of important use teams in the workplace. Creating a highly functional team means more than just handing out tasks to a random group of people; This requires prior understanding and an effective approach.

One of the eternal questions of leaders is: What makes a High-Performance organization? Numerous companies claim to be high quality organizations. It is actually rare but instantly recognizable.

High quality organizations are role models for the organizational world. They represent real versions of the modern managerial ideal: an organization that excels in these many areas that it consistently outperforms most of its competitors over a long period of time.

Leading a high performance organization teaches and assists employees to achieve better results by supporting, assisting them, protecting them from cu2(es)[( )]TETrx 9tses T0 0 0 rgvcc( )-2,(n)-3(s5

What is high performance workplace

To explain what a highly performance workplace is - it is a physical or virtual environment designed to make employees as efficient as possible in achieving business goals and values. A highly

Why is important high performance workplace

Business owners need employees who can do the job assigned to them, because the activities and behaviors of each employee determine the success of the company. It is a highly qualified environment that makes employees more trusting of each other, more independent in some matters and make their own decisions, and satisfied with their work. All this is necessary for employees to be more responsible, to work more productively, which is essential for the company, because these factors play an important role in the success of a business. High performance workplace have become very important as a source of competitive advantage in today's competitive business environment. Human resource management capabilities are important for attracting, selecting, retaining, motivating and developing the workforce in the organization. Meanwhile, organizational culture, which is viewed as organizational capital, can also be a driver of sustainable competitive advantage. Organizational values, as a reflection of the culture of the organization, asserts strategic issues such as strategic change, management decision-

3 most important things that make

of effective communication; there can be trust between team members only if they are allowed to air their views freely. This is the reason why organizations often undertake team-building exercises that put team members in positions of trust.

Little overview about performance workplace in Georgian organizations

Today, unfortunately, most Georgian companies do not consider it necessary to develop a highly performance workplace. They have staff and they think that these staff no longer need development, retraining, training. Their main problem is that paying for all this is a pointless expense, but they do not realize that the cost now will bring more profit and success to the company in the future.

If I take examples from my personal work experience, the companies where I worked nowhere paid attention to the above issue. According to the top management, the employees

All this is necessary to achieve an effective working environment, which is related to the

Utility maximization problem with uncertainty of success probability

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Tsotne Kutalia, Revaz Tevzadze

Abstract

Abstract. We study a robust portfolio optimization problem under model uncertainty for an investor with logarithmic or power utility.

1 Problems formulation

Let S be stock price defined by

$$S_{+1}(!_1 ::: ! \ !_{+1}) \quad (1_{(\ n+1 = \ )} u + 1_{(\ n+1 = \ )} d) S \ (!_1 ::: ! \ );$$

$! \quad !_1 ::: ! \quad 2 \ fH; Tg \ ; \ \tilde{q} \quad \frac{1}{\ }; \ \tilde{p} \quad \frac{1+}{\ }: \ r$ is interest rate. Evidently that $S \quad S_0 u^{(\ )} d^{(\ )} \quad S_0 d \ (\frac{}{})^{(\ )}$. Let $\tilde{P}(!) \quad \tilde{p}^{(\ )}(1 \quad \tilde{p})^{(\ )}$ be risk-neutral probability measure on $fH; Tg$ and $P(!) \quad p^{(\ )}(1 \quad p)^{(\ )}$ is reference measure. For

$$Z(p) \quad \frac{P}{\tilde{P}} \quad \frac{p}{\tilde{p}}^{(\ )} \quad \frac{1 \quad p}{1 \quad \tilde{p}}^{(\ )} \quad \frac{p}{\tilde{p}} \quad \frac{q}{\tilde{q}} \quad \frac{p\tilde{q}}{\tilde{p}q} \quad \frac{q}{\tilde{q}}$$

we have

$$Z(p) \quad g \quad \frac{S}{S_0} \quad ;$$

where $g(x) \quad \left(\frac{\cdot}{\ \log_{u/d}(p/q)}\right. \quad x^{\log_{u/d}(\ \cdot \ )}$.

It is well known that utility maximization problem

$$EU(X \ );$$

for wealth process $X_{+1} \quad \Delta \ S_{+1} + (1 + r)(X \quad \Delta \ S \ )$ is equivalent to

$$EU(V); \ X_0^0 \quad X_0(1 + r) \ :$$

The solution of utility maximization problem

$$\underset{\tilde{=}_0}{} EZ(p)U(V)$$

when p is known exactly and U is either the logarithmic utility $U(x) \quad 1 \; (x)$ or a power utility $Ux) \quad \frac{1}{}x \;; \quad < 1; \quad 6 \; 0$ is expressed explicitly and equal to

$$V \quad X_0^0 \frac{Z(p)^{\frac{1}{1-\gamma}}}{\tilde{E}Z(p)^{\frac{1}{1-\gamma}}}; \quad < 1:$$

Our aim is to investigate the problem

$$\underset{:\,\tilde{}\;=(1+.\,)^N \;\; 0}{} \; \underset{2\Theta}{} \tilde{E}Z(p)U(V); \tag{1}$$

$$\text{where } Z(p) \quad \frac{p}{\tilde{p}}^{\;(\;)} \quad \frac{1 \quad p}{1 \quad \tilde{p}}^{\;(\;)} \quad ; \; \Theta \quad 0; 1 \, :$$

Finally the optimal V has to hedged, i,e. it will exists $X_0; X_1; :::; X$ such that $X \quad V$.

The general cases of Θ we consider are $fp_0; p_1g; \;\; p_0; p_1 \;; \;\; p_0; p_1 \; n(\tilde{p} \quad "; \tilde{p}+ "); \; p_0 < p_1$. In the sequel we use the notation $P(\Theta)$ for set of probability measures on Θ and $Z(\;) \quad {}_{\Theta} Z(p) \; (dp)$. Also we use the notation $Z \quad Z(p_0) + (1 \quad)Z(p_1); \quad 2 \;\; 0; 1$ for the $Z(\;); \qquad {}_0 + (1 \qquad) \;\; {}_1$. Since

$$\underset{2\Theta}{} \tilde{E}Z(p)U(V) \qquad \underset{2P(\Theta)}{} \tilde{E}Z(\;)U(V);$$

the problem (1) we can change by

$$\underset{2P(\Theta)}{} \; \underset{:\,\tilde{}\;=(1+.\,)^N \;\; 0}{} \tilde{E}Z(\;)U(V); \tag{2}$$

which allows us to study at first

$$\underset{:\,\tilde{}\;=(1+.\,)^N \;\; 0}{} \tilde{E}Z(\;)U(V):$$

$$\frac{1}{}X_0^0 \; \tilde{E}Z(\;) \frac{Z(\;)^{\frac{\gamma}{1-\gamma}}}{\tilde{E} \;\; Z(\;)^{\frac{1}{1-\gamma}}} \qquad \frac{1}{}X_0^0 \; \tilde{E}^1 \quad Z(\;)^{\frac{1}{1-\gamma}}; \quad < 1:$$

Hence it remains to solve

$$\tilde{E}^1 \quad Z(\;)^{\frac{1}{1-\gamma}}; \quad 6 \;\; 0$$

or

$$\tilde{E}Z(\;) 1 \;\; Z(\;); \qquad 0$$

2 The uncertainty of success probability

Let Θ $fp_0; p_1g$ and $_0 + (1 \quad) \; _1$. It needs to find $_0 \quad _1 f(\;)$, where

$$f(\;) \quad \begin{matrix} \vec{E}^1 \quad Z(\;)^{\frac{1}{1-\gamma}}; \quad 6 \; 0 \\ \vec{E} Z(\;) 1 \quad Z(\;); \qquad 0: \end{matrix}$$

Proposition 1. Let be a minimum point of function $f(\;) \quad \vec{E} Z \; 1 \; Z$ $E \; 1 \; Z \; ; \; i:e: f(\quad) \qquad f(\;)$. Then

$$\begin{matrix} 1; \; if \; p_0 < p_1 \quad \tilde{p}; \\ f^{0 \; \; 1}(0) \text{ (i.e. the unique solution of } f^0(\quad) \quad 0); \; if \; p_0 < \tilde{p} < p_1; \\ 0; \; if \; \tilde{p} \quad p_0 < p_1: \end{matrix}$$

Moreover

$$\begin{matrix} f^0(0) < 0; \; f^0(1) > 0; \; if \; p_0 < \tilde{p} < p_1; \\ f^0(\;) \quad 0; \quad 2 \; 0; 1 \; ; \; if \; p_0 < p_1 \quad \tilde{p}; \\ f^0(\end{matrix}$$

Since

$$C\cdot\ (p_0q_0\ \cdot\ \ p_1q_1\ \cdot\ )\,\mathrm{l}\ \ (\vec{p})\cdot\ (\vec{q})\ \cdot \quad {}_{\cdot\,=0}$$

$$\mathrm{l}\ \ (\vec{p})\ {}_{\cdot\,=0}\ kC\cdot\ (p_0q_0\ \cdot\ \ p_1q_1\ \cdot\ ) + \mathrm{l}\ \ (\vec{q})\ {}_{\cdot\,=0}\ (N\ \ k)C\cdot\ (p_0q_0\ \cdot\ \ p_1q_1\ \cdot\ )$$

$$\mathrm{l}\ \ (\vec{p})\ {}_{\cdot\,=0}\ kC\cdot\ (p_0q_0\ \cdot\ \ p_1q_1\ \cdot\ )\quad \mathrm{l}\ \ (\vec{q})\ {}_{\cdot\,=0}\ kC\cdot\ (p_0q_0\ \cdot\ \ p_1q_1\ \cdot\ )$$

$$\mathrm{l}\ \ \frac{\vec{q}}{\vec{p}}\ {}_{\cdot\,=1}\ kC\cdot\ (p_0q_0\ \cdot\ \ p_1q_1\ \cdot\ )\quad \mathrm{l}\ \ \frac{\vec{q}}{\vec{p}}\ {}_{\cdot\,=1}\ NC\cdot\ {}^{1}_{1}(p_0q_0\ \cdot\ \ p_1q_1\ \cdot\ )$$

$$\mathrm{l}\ \ \frac{\vec{q}}{\vec{p}}\ {}_{\cdot\,=1}\ Np_0C\cdot\ {}^{1}_{1}p_0\ {}^{1}q_0\ \cdot\ \ {}_{\cdot\,=1}\ Np_1C\cdot\ {}^{1}_{1}p_1\ {}^{1}q_1\ \cdot$$

$$\mathrm{l}\ \ \frac{\vec{q}}{\vec{p}}\ {}_{\cdot\,=1}\ NC\cdot\ {}^{1}_{1}(p_0q_0\ \cdot\ \ p_1q_1\ \cdot\ )$$

$$\mathrm{l}\ \ \frac{\vec{q}}{\vec{p}}\ {}_{\cdot\,=1}\ Np_0C\cdot\ {}^{1}_{1}p_0\ {}^{1}q_0\ \cdot\ \ {}_{\cdot\,=1}\ Np_1C\cdot\ {}^{1}_{1}p_1\ {}^{1}q_1\ \cdot$$

$$(Np_0\ \ Np_1)\,\mathrm{l}\ \ \frac{\vec{q}}{\vec{p}}\ ;$$

we have

$$f^0(\ )\ {}_{\cdot\,=0}\ C\cdot\ (p_0q_0\ \cdot\ \ p_1q_1\ \cdot\ )\,\mathrm{l}\ \Big(\ p_0q_0\ \cdot + (1\ \ )p_1q_1\ \cdot\ \Big)$$

$$N(p_0\ \ p_1)\,\mathrm{l}\ \ \frac{\vec{q}}{\vec{p}}\ :$$

Hence

$$f^0(1)\ {}_{\cdot\,=0}\ C\cdot\ (p_0q_0\ \cdot\ \ p_1q_1\ \cdot\ )\,\mathrm{l}\ \ (p_0)\cdot\ (q_0)\ \cdot$$

$$N(p_0\ \ p_1)\,\mathrm{l}\ \ \frac{\vec{q}}{\vec{p}}$$

$$N(p_0\ \ p_1)\,\mathrm{l}\ \ \frac{p_0}{q_0}\quad N(p_0\ \ p_1)\,\mathrm{l}\ \ \frac{\vec{q}}{\vec{p}}\quad (Np_0\ \ Np_1)\,\mathrm{l}\ \ \frac{p_0\vec{q}}{q_0\vec{p}}\ :$$

and

$$f^0(0)\ {}_{\cdot\,=0}\ C\cdot\ (p_0q_0\ \cdot\ \ p_1q_1\ \cdot\ )\,\mathrm{l}\ \ (p_1)\cdot\ (q_1)\ \cdot$$

$$(Np_0\ \ Np_1)\,\mathrm{l}\ \ \frac{p_1\vec{q}}{q_1\vec{p}}$$

From $p_0 < \tilde{p} < p_1$ follows $\frac{0 \tilde{}}{0 \tilde{}} < 1$ and $\frac{1 \tilde{}}{1 \tilde{}} > 1$ (since $\frac{0 + 0}{0}$ $\frac{1}{0} > \frac{1}{\tilde{}}$ $\frac{\tilde{} + \tilde{}}{\tilde{}}$) $\frac{0}{0} > \frac{\tilde{}}{\tilde{}}$). Thus $f'(1) > 0; \ f'(0) < 0$.

If $p_0 < p_1$ $\tilde{p}$ then from $f'(0) > 0$ and $f''(\ )$ $\tilde{E}(Z(p_0)$ $Z(p_1))^2 \frac{1}{\alpha} > 0$ follows $f'(\ ) > 0;$ 2 $0; 1$. Similarly one can prove $f'(\ ) < 0; \ 8$, if $\tilde{p}$ $p_0 < p_1$.

Corollary. If $\tilde{p}$ $\tilde{q}$ $\frac{1}{2}$ and p_1 1

More general for $\tilde{} P \quad \tilde{}_{( )} \, (\# \; ( ) (1 \quad) \# \; ( ))$ the similar equality

$$Z(\tilde{} P \,) \qquad Z(\tilde{} (x)) is E \; / \; is S h$$
$$2\Omega$$

A Martingale Characterization of the General Solution of Quadratic Functional Equation

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A. We describe the class of functions $f = (f(x), x \in R)$, for which the process $f(W_t) - Ef(W_t)$ is a martingale. This result is applied to give a martingale characterization of the general solution of the quadratic functional equation.

1. I. It is well known that if for a function $f = (f(x), x \in R)$ the transformed process $(f(W_t), t \ge 0)$ of a Brownian Motion W is a martingale, then f is a linear function (see, e.g. [2]). In this paper we describe the class of function f for which the process $f(W_t) - Ef(W_t)$ is a martingale. We prove that the process

$$f(W_t) - Ef(W_t), \quad t \ge 0$$

is a right-continuous martingale if and only if the function f is a square trinomial , i.e., $f(x) = ax^2 + bx + c$ for some constants a, b and c. Besides, we show that if $f(W_t) - Ef(W_t)$ is a martingale (without assuming the regularity of paths), then $f(x)$ is equal to some square trinomial almost everywhere with respect to the Lebesgue measure.

We use this result to characterize solutions of quadratic functional equation

$$f(x+y) + f(x-y) = 2f(x) + 2f(y) \quad \text{for all} \quad x, y \in R; \qquad (1)$$

the general measurable solution of which (see, e.g., [5]) is the function $f(x) = ax^2; a \in R$. We give an equivalent characterization of equation (1) in terms of martingales, which gives also a probabilistic proof of this assertion.

2. . Let $W = (W_t; t \geq 0)$ be a standard Brownian Motion defined on a probability space $(\Omega; F; P)$ with filtration $F = (F_t; t \geq 0)$ satisfying the usual conditions of right-continuity and completeness.

1. Let $f = (f(x); x \in R)$ be a function of one variable, such that $f(W_t)$ is integrable for every $t \geq 0$.

a) If the process

$$M_t = f(W_t) - Ef(W_t); \quad t \geq 0;$$

is a right-continuous (P- a.s.) martingale, then the function f is of the form

$$f(x) = ax^2 + bx + c \quad \text{for some constants} \quad a; b \quad \text{and} \quad c \in R. \qquad (2)$$

b) If the process M is a martingale, then $f(x)$ coincides with the function $ax^2 + bx + c$ (for some constants $a; b; c \in R$) almost everywhere with respect to the Lebesgue measure.

Proof. a) Let

$$g(t) \equiv Ef(W_t) = \int_R f(y) \quad 1x$$

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$P - a.s.$ for all $t \le T$. Let

$$u(t, x) = E(f(W_T)/W_t = x).$$

It is well known that $u(t, x)$ will be of the class $C^{1,2}$ on $(0, T) \times R$ and satisfies the backward Kolmogorov equation (see, e.g. [3])

$$\frac{\partial u}{\partial t} + \frac{1}{2}\frac{\partial^2 u}{\partial x^2} = 0, \quad 0 < t < T, x \in R. \tag{4}$$

By the Markov property of W

$$u(t, W_t) = E(f(W_T)/F_t)$$

and from (3) we have that P-a.s.

$$f(W_t) = u(t, W_t) - g(T) + g(t).$$

Therefore,

$$\int_R \left| f(x) - u(t, x) + g(T) - g(t) \right| \frac{1}{\sqrt{2\pi t}} e^{-\frac{x^2}{2t}} dx = 0$$

which implies that for any $0 < t \le T$

$$f(x) = u(t, x) - g(T) + g(t) \quad a.e. \tag{5}$$

The solutions of these equations are

$$f(x) = ax^2 + bx + c \quad \text{and} \quad g(t) = at + c \tag{9}$$

respectively, for some $a, b, c \in R$.

b) Let

$$\tilde f(x) = u(t_0, x) + g(t_0) - g(T).$$

for some $t_0 > 0$. It follows from (5) that

$$\lambda(x : f(x) \neq \tilde f(x)) = 0, \tag{10}$$

where λ is the Lebesgue measure and by definition of $u(t, x)$ the function $\tilde f(x)$ is continuous (moreover, it is two-times differentiable). It follows from (10) that $P(f(W_t) = \tilde f(W_t)) = 1$ for any $t \geq 0$ and since $Ef(W_t) = E\tilde f(W_t)$, we obtain that the processes $M_t = f(W_t) - Ef(W_t)$ and $\tilde M_t = \tilde f(W_t) - E\tilde f(W_t)$ are equivalent, which implies that the process $\tilde M_t$ is a continuous martingale. Therefore, it follows from part a) of this theorem that $\tilde f(x)$ is of the form (2) and hence, $f(x)$ coincides with the function (2) almost everywhere with respect to the Lebesgue measure. □

. The converse is also true. If the function f is of the form (2), then

$$f(W_t) = aW_t^2 + bW_t + c, \quad Ef(W_t) = at + c$$

and the process $f(W_t) - Ef(W_t) = a(W_t^2 - t)$ is a martingale.

1. Let $f = (f(x), x \in R)$ be a function of one variable.

a) If the process $(f(W_t), \mathcal{F}_t, t \geq 0)$ is a right-continuous martingale, then

$$f(x) = bx + c \quad \text{for all} \quad x \in R \tag{11}$$

for some constants b, c.

b) If the process $(f(W_t), \mathcal{F}_t, t \geq 0)$ is a martingale, then $f(x) = bx + c$ almost everywhere with respect to the Lebesgue measure for some constants $b, c \in R$.

Proof. If the process $f(W_t)$ is a martingale, then $g(t) = Ef(W_t)$ is constant and the coefficient a in (9) is equal to zero. Therefore, this corollary follows from Theorem 1. □

3 A . It was proved in [4] that if the function $f = (f(x); x \in R)$ is a measurable solution of the Cauchy additive functional equation

$$f(x + y) = f(x) + f(y); \quad \text{for all} \quad x; y \in R;$$

then the transformed process $(f(W); t \quad 0)$ is a right-continuous martingale, which (by Corollary 1 of Theorem 1) implies that f is a linear function. Here we propose a similar characterization of solutions of the quadratic functional equation

$$f(x + y) + f(x \quad y) = 2f(x) + 2f(y) \quad \text{for all} \quad x; y \in R: \tag{12}$$

It is well known (see, e.g., [5]), that the general measurable solution of equation (12) is the function $f(x) = ax^2; a \in R$. Using Theorem 1 we give an equivalent characterization of equation (12) in terms of martingales , which gives also a probabilistic proof of this assertion.

2. Let the function $f = (f(x); x \in R)$ be non-negative (or non-positive) for all $x \in R$. The following assertions are equivalent:

i) the function $f = (f(x); x \in R)$ is a measurable solution of (12),

ii) $f = (f(x); x \in R)$ is an even function with $f(0) = 0$ and such that the process

$$N = f(W) \quad Ef(W); \quad t \quad 0;$$

is a right-continuous martingale,

iii) the function f is of the form

$$f(x) = ax^2; \tag{13}$$

for some constant $a \in R$.

Proof. $i) \; ! \; ii)$ It is evident that if f is a solution of (12) then $f(0) = 0$ and $f(x) = f(\quad x)$ for all $x \in R$. Therefore, since $f(x)$ is non-negative or non-positive, expectations below have a sense and for any random variable with symmetric distribution

$$Ef(\quad x) = Ef(x \quad) = Ef(\quad + x): \tag{14}$$

Substituting $x = W \quad W$ and $y = x + W$ in equation (12) we have that

$$f(W) + f(W \quad 2W \quad x) = 2f(W \quad W) + 2f(x + W): \tag{15}$$

Since $E(W_t - 2W_s)^2 = t$ for $s \le t$, the random variables W_t and $W_t - 2W_s$ have the same normal distributions and hence

$$Ef(x + W_t) = Ef(x + 2W_s - W_t) = Ef(W_t - 2W_s - x). \qquad (16)$$

Therefore, taking expectations in (15) we obtain

$$Ef(x + W_t) = Ef(W_t - W_s) + Ef(x + W_s) \quad s \le t. \qquad (17)$$

Let $g(t) = Ef(W_t)$. Then $Ef(W_t - W_s) = g(t - s)$ and it follows from (17) (for $x = 0$) that g satisfies the Cauchy additive functional equation

$$g(t) = g(t - s) + g(s); \quad s \le t$$

on R^+. As it is well known (see, e.g., [1]) that any bounded from bellow (or from above) solution of this equation is of the form $g(t) = ct$ for some constant $c \in R$. Therefore, $f(W_t)$ is integrable for any $t \ge 0$ and if we take $s = 0$ in (17) we obtain

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$$f(x) = Ef(x + W_t) - Ef(W_t) = \int_R f(x + y) \frac{1}{\sqrt{2\pi t}} e^{-\frac{y^2}{2t}}$$

$f(x + 1$

follows from equations (19) and (20). Thus, the process $N = (f(W) - Ef(W); t \geq 0)$ is a martingale with P-a.s. continuous paths.

$ii) \to iii)$. Since $f(W) - Ef(W)$ is a martingale, Theorem 1 implies that the function f should be of the form

$$f(x) = ax^2 + bx + c; \quad a; b; c \in R.$$

Since f is even we have that $b = 0$ and $c = 0$ since $f(0) = 0$. Thus, $f(x) = ax^2$ for some $a \in R$.

The proof of implication $iii) \to i)$ is evident.

□

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MhXW! mQmZR! kVRihZR! X_mR^\ZW! \Q]VZ! kR^_! RZm]XnQiXYa! mR^iZniX\Z! mVi! WiRQiZav! `v! \QRvXYa! ihZ! WiRXsZ! mRXnZ! ^k! ihZ! mVi! ^miX^Y! Qnn^RTXYa! i^! a^]T! mRXnZ! SZZs]v! mRZTXniX^YW! `v! _QnhXYZ!]ZQRYXYa! _Zih^TW8! MhXW5! XY! iVRY5! RZTVnZW! n^Wi! ^k! RZ`Q]QYnXYa! ihZ! m^Rik^]X^5! X8Z85! XW! ihZ! n^Wi! ^k! ihZ! mVi! ^miX^Y! XiWZ]k! QYT5! kXYQ]]v5! ihZ! i^iQ]! n^Wi! ^k! ihZ! m^Rik^]X^! ihQi! XW! VWZT! QW! ihZ! WiRXsZ! mRXnZ! ^k! ihZ! RZQ]! ^miX^Y! XY! ihZ! XY\ZWi_ZYi! mR^wZni8!

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xQ]VZ! ^k! mR^iZniX\Z! mVi! SXih! kXyZT! Qi=ihZ=_^YZv! WiRXsZ! mRXnZ!

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MhXW! WZniX^Y! k^]]^SW! mRQniXnQ]! ZyQ_m]Z! TZWnRX`ZT! XY! tHGu! SXih! zZ^RaXQY! mR^TVnZR! ^k! a^]T! SXTaZiW5! {QRQmshQYQ! QYT! XiW! a^]T! mRXnZ! RXWs! hZTaXYa! mR^`]Z_8! |mm^WZT! i^! [^`VWi! xQ]VQiX^Y! iZnhYX}VZW! TZWnRX`ZT! XY! tHu! QYT! tHGu5! ShZRZ! mR^wZni! \Q]VQiX^Y! XW! RZ}VXRZT! VYTZR! XYn^_m]ZiZ! _QRsZiW! WnZYQRX^! QYT! Xi! XW! k^nVWZT! ^Y! RXWs! _XiXaQiX^Y! ^k! Y^Y=iRQTQ`]Z! QWWZiW! WVnh! QW! SXTaZiW! mR^TVnZT! `v!]^nQ]! _QYVkQniVRZRW5! ihXW! mQmZR! k^nVWZW! ^Y! RXWs! _QYQaZ_ZYi! ^k! z^]T! mRXnZ! RXWs! QW! QY! XYmVi! n^Wi! k^R! mR^TVnXYa! SXTaZiW8! UQWZT! ^Y! WiQiXWiXnQ]! QYQ]vWXW! QYT! ~^YiZ! •QR]^! qX_V]QiX^Y! TXWnVWWZT! XY! t:u! QYT! WQ_m]Z! TQiQ! XY! tHeu!

•^R!WX_m]XnXiv5!]Zi!TXWn^VYi!RQiZ5!R5!Z}VQ]!i^!G<!mZR!vZQR8! , YTZR!^YZ=vZQR!mZRX^T!XY\ZWi_ZYi!
h^RXl^Y!mRZWZYi!\Q]VZ!BpxD!^k!nQWh!k]^S!kR^_!SXTaZi!mR^TVniX^Y!XW!aX\ZY!QWc!

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oZyi!XY\ZWi_ZYi!\Q]VZ!^k!ihZ!mR^wZni!XW!ZyiZYTZT!i^!n^\ZR!ihZ!^mm^RiVYXiv!i^!hZTaZ!ihZ!a^]T!
mRXnZ! SXih! mVi! ^miX^Y! VWXYa! [ZQ]! |miX^YW! ƒ YQ]vWXW! TXWnVWWZT! XY! tGu5! tKu5! QYT! t?u8

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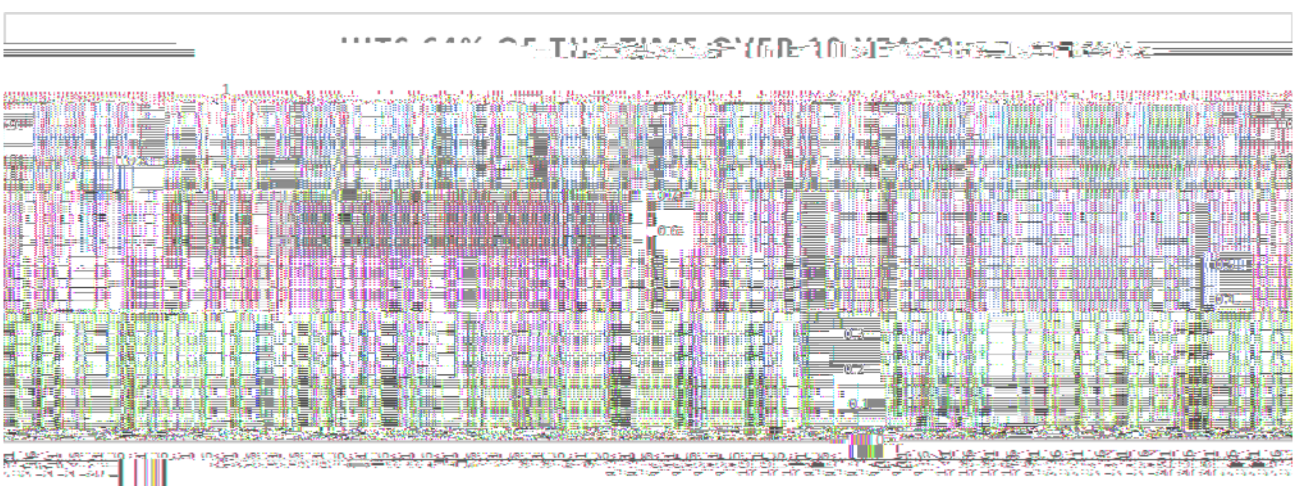
,]iX_QiZ]v5! ^miX_Q]! WiRXsZ! mRXnZ! SX]]! TZmZYT! ^Y! ihZ! kViVRZ! a^]T! mRXnZ8! LY!]XYZ! SXih! SZZs]v! RZ`Q]QYnXYa!^k!mR^iZniX\Z!mVi!m^Rik^]X^5!^YZ!nQY!n^_Z!Vm!SXih!SZZs]v!VmTQiZT!mRZTXniX^YW!^k! a^]T!mRXnZ!QYT!VWZ!ihQi!QW!Q!WiRXsZ!mRXnZ!^k!ihZ!mVi8!LY!ihXW!WZniX^Y5!`QWXn!~QnhXYZ!„ZQRYXYa! B~„D!~^TZ]W!kR^_!tH>u!WVnh!QWc!

= …=YZQRZWi!YZXah`^R!B…o oD!
= zRQTXZYi!U^^WiXYa!MRZZW!BzUMD!
= qVmm^Ri!xZni^R!~QnhXYZW!Bqx~D!
= zQVWWXQY!pR^nZWWZW![ZaRZWWX^Y!Bz[pD!

ƒaQXY5!VWXYa!WQ_m]Z!TQiQ!XY!tHeu5!ZYWZ_`]Z!^k!Q`^\Z!~„!_^TZ]W!XW!VWZT!i^!mRZTXni!SZZs]v! mRXnZW!^k!a^]T8!ƒY!ZyQ_m]Z!^k!WVnh!mRZTXniX^YW!n^_mQRZT!SXih!QniVQ]!a^]T!mRXnZW!^\ZR!mZRX^T! ^k!H;!vZQRW!XW!X]]VWiRQiZT!`Z]^Sc!



ƒ`^\Z!aRQmh!Wh^SW!ihQi!>?<!^k!ihZ!iX_Z!mRZTXniX^YW!SZRZ!QnnVRQiZ!Q`^Vi!ihZ!TXRZniX^Y!^k! a^]T!mRXnZ!XYnRZQWZ!^R!TZnRZQWZ8!LY!ihZ!YZyi!WZniX^Y5!SZZs]v!mRZTXniX^YW!^k!a^]T!mRXnZ!SX]]!`Z! VWZT!QW!ihZ!WiRXsZ!mRXnZ!k^R!mVi!^miX^Y!XY!hZTaXYa!WiRQiZav8!

xQ]VZ!^k!mR^iZniX\Z!mVi!SXih!\QRXQ`]Z!WiRXsZ!mRXnZ!

!

o^S5!XiW!X_m^RiQYi!i^!n^_mQRZ!h^S!SZ]]!mRZTXniZT!a^]T!mRXnZ!S^Rs!QW!Q!\QRXQ`]Z!WiRXsZ!mRXnZ! k^R!mR^iZniX\Z!mVi!WiRQiZav!QaQXYWi!kXyZT!WiRXsZ!mRXnZ8!•R^_!ZyQ_m]Z!XY!tHeu5!i^iQ]!SZQ]ih!kR^_! mR^iZniX\Z!mVi!WiRQiZav!QYT!a^]T!mRXnZ!WX_V]QiX^Y!TXWnVWWZT!XY!t:u!XW!Wh^SY!`Z]^Sc!

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•VRihZR!mR^nZZTXYa!SXih!RZQ]!^miX^Y!\Q]VQiX^Y!^k!ihXW!XY\ZWi_ZYi!mR^wZni5!QTTXiX^YQ]!XYnRZQWZ!
XY!^miX^YQ]!\Q]VZW!XW!aQXYZT!XY!Q_^VYi!^k!HK8?G!mZR!aRQ_8!•XYQ]!XY\ZWi_ZYi!mR^wZni!\Q]VZ!XW!
aX\ZY!`vc!

[ZkZRZYnZW!

tHu! [^`VWi! ~ZQY=xQRXQYnZ! PZTaXYa! ƒYT! pRXnXYa! ^k! •^YiXYaZYi! •]QX_W! LY! ƒ! | YZ! pZRX^T! ~^TZ]! BG;HHD! f! *[8! MZ l lQT l Z! QYT! M8! , lVYQqh lX]X!* f! LYiZRYQiX^YQ]! r^VRYQ]! ^k! MhZ^RZiXnQ]! QYT! ƒmm]XZT! •XYQYnZ! f! g ^R]T! qnXZYiXkXn! pV`]XWhXYa! •^_mQYv!!

tGu! [ZQ]! | miX^YW! ƒYQ]vWXWc! i^^]W! QYT! iZnhYX}VZW! k^R! \Q]VXYa! WiRQiZaXn! XY\ZWi_ZYiW! QYT! TZmXXmmV!f]XWhZ*ThQY! ~VY*!fGYT!ZTXiX^Y5!mV`]XWhZT!`v!r^hY! g X]Zv! †!qXYW5!LYn85!G;;>!

tKu! LY\ZWi_ZYi! pR^wZniW! xQ]VQiX^Y! ,WXYa! [ZQ]! | miX^YW! QYT! zQ_Z! MhZ^Rv! BG;HKD! f! *„Z l QY! zQnhZnhX]QT l Z! QYT! LRQs]X! •hZ]XT l Z*! f! ƒnQTZ_Xn=QYQ]viXnQ]! r^VRYQ]! ‡^n^Y^_XnW! QYT! UQYsXYa‰! G;HK! x^]8! L5! oK!

t?u! •^y! Zi! Q]8! BH:Š:D! f! *•^yy5! [^WW5! [V`XYWiZXY*!

teu! [^`VWiYZWW! BG;;‹D! f! *„QRW! pZiZR! PQYWZY5! Mh^_QW! r8! qQRaZYi*! f! pV`]XWhZT! `v! pRXYnZi^Y! ,YX\ZRWXiv! pRZWW!

t>u! PZTaXYa! !T91f5O1OTm[(Q)O.1(Y)O.1(Q)O.1(])-O.1(v)-O.1(i)-O.1(X)-O.2(n)O.2(Q)O.