As can be seen from Figure 1, the total assets of the Halyk Bank is around 3 times larger than that of Kaspi Bank, however the market capitalization figures show the rather opposite picture. Namely, the market cap of Kaspi Bank is about 6.4 times higher than that of Halyk Bank. This difference is a result of the digital disruption initiated by Kaspi Bank in the Kazakhstan banking Sector.

Fig.2 Comparison of the Equity and Market Capitalization

The effect of digital disruption is even more evident when comparing the equity size with the market cap for these two banks. It can be seen from Figure 2 that the market cap of Halyk Bank is almost the same as its equity, while the market cap of Kaspi Bank is more than 30 times its equity t a truly remarkable result. This is the result of the market expectation for higher returns due to synergy effects of an innovative digital ecosystem set up by Kaspi Bank, which incorporates into the ecosystem the banking services with Fintech solutions, electronic marketplace and loyalty platform in association with a large number of merchants.

In order to evaluate the effec

In contrast

Fig.4 Comparison of the Operating Profit and Asset Turnover

The next step in our analysis will be the identification of the reasons behind the 2.28 times higher asset turnover of Kaspi Bank in comparison to Halyk Bank. This analysis is presented in Figure 5, which gives the breakdown of the asset turnover in terms of interest revenues and fees & commissions

The analysis of the financial results presented in Figure 5 shows that the interest revenue per asset of Kaspi Bank is 1.63 times higher than that of Halyk Bank. This can be logically attributed to the more retail-oriented portfolio of Kaspi Bank and to a lesser accent on mortgage loans that are traditionally characterized with lower APRs.

However, the more prominent picture emerges

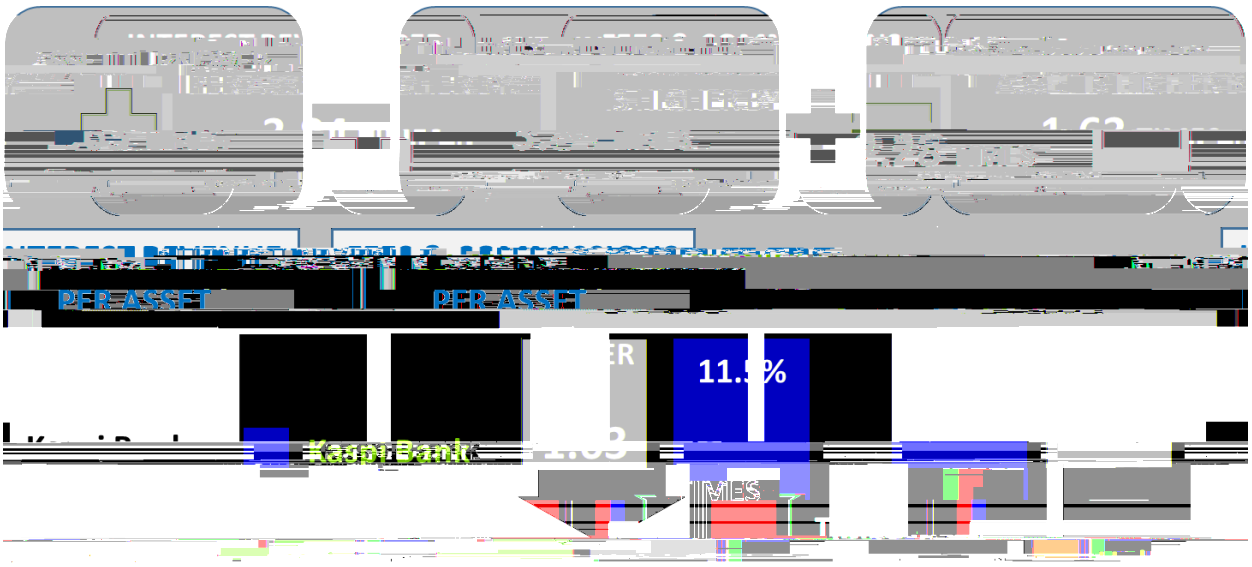

Fig. 5 Breakdown of the asset turnover into interest revenues and fees & commissions

Even more amazing is the fact that the revenue of Kaspi Bank generated from fees & commissions is almost exactly equal to its interest income (as shown in Figure 5), which is an extremely rare case in the banking industry. This fact indicates on the effectiveness of cross-sectoral cooperation and the synergy effects derived from such cooperation. The result is the high profitability of the ecosystem delivering full-scale digital services to large segments of customers in a convenient and comfortable manner.

Fig 6

Fig. 6 Benefits of a Digital Ecosystem Model

A Generalization of the von Bertalan y growth Model using the BSDE Approach

with initial condition L(t

If the process X cannot make jumps larger than 1 (which is natural to assume in this context), then solution of this equation is

$$L_t = L_1 (1 \quad E_t(X)); \tag{5}$$

where $E_t(X)$ is the stochastic exponent of the process X and the extreme length L_1 is assumed to be a constant. Note that L_t de ned by this model is increasing process and it coincides with the von Bertalan y growth curve when X is a deterministic subordinator $X_t = kt$.

This approach (as all existing) has a drawback as a growth model, since the asymptotic length of the sh is assumed to be a constant. This implies that the variation of sh length tends to zero, which is not realistic, as it would imply that all individuals should reach the same limiting size. In order to overcome this problem it seems natural to assume that the extremal size of a sh is itself a random variable, thus accounting for the individual variability. Therefore, it is natural to use Backward SDE's (instead of the forward SDEs) with the random boundary condition at the end equal to the asymptotic length of a sh.

To generalize the von Bertalan y model when the extreme length L_1 is a random variable, let rst consider the simple case and only assume that L_1 is a bounded random variable measurable with respect to $F_1^W = \__{t\,0} F_t^W$, where W is a Brownian Motion and $(F_t^W; t \quad 0)$ is the ltration generated by W.

We write this model as a solution of the Backward Stochastic Di erential equation (BSDE)

$$Y_t = \int_0^t Y_s \frac{K e^{Ks}}{1 \quad e^{Ks}} ds + \int_0^t Z_s dW_s; \tag{6}$$

with the boundary condition

$$Y_1 = \lim_{t!1} Y_t = L_1 : \tag{7}$$

The solution process to equation (6)-(7) is

$$L_t = E(L_1 jF_t^W)[1 \quad e^{Kt}]: \tag{8}$$

More exactly the solution of (6)-(7) is a pair $(Y_t; Z_t)$

$$Y_t = L_t; \quad Z_t = '_t(1 \quad e^{Kt});$$

where L_t is de ned by (8) and $'_t$ is the integrand from the integral representation of the martingale

$$E(L_1 jF_t^W) = EL_1 + \int_0^t '_s dW_s;$$

which can be immediately veri ed by the integration by part formula.

Note that, since (8) implies

$$EL_t = EL_1 [1 \quad e^{Kt}];$$

the expectation of L_t follows the Von Bertalan y-type pattern with L_1 replaced by EL_1.

Remark that, if in (8) instead of exponential distribution function $1 \quad e^{Kt}$ we shall take general continuous distribution function $G(t)$, then the process $L_t = E(L_1 jF_t^W)G(t)$ will satisfy the BSDE

$$Y_t = \int_0^t \frac{Y_s}{G(s)} dG(s) + \int_0^t Z_s dW_s; \qquad (9)$$

with the same boundary condition (7).

We shall generalize expression (5) (see Theorem 1) assuming that L_1 is a random variable and consider this variable as a boundary condition at in nity of a BSDE for L_t driven by a subordinator X and a Brownian Motion W, independent of X. The linear BSDEs derived in the paper di er from classical cases by considering not integrable coe cients on the in nite time interval. Under additional assumption that the extreme size L_1 of a sh is a random variable measurable with respect to the -algebra F_1^W generated by the Brownian Motion W, i.e., when two sources of randomness, the random individual variability (related with L_1) and the environmental randomness (related with the process X_t), are independent, the BSDE takes simpler and more natural form (see Corollary 1).

2 The main results.

Let $X = (X_t; t \quad 0)$ be a Levy process with a ne processt; $> \quad 0$, with zero Brownian part and with positive jumps (a subordinator). Let

$W = (W_t$

De nition. Let V be the class of cadlag processes X_t $(t \geq 0)$, such that the family of random variables $(Y; \in T)$ is uniformly integrable, where T is the set of stopping times.

Theorem 1. Let X be a Levy process with increasing paths (a subordinator) and let $X_t < 1$ for all $t \geq 0$. Assume that L_1 is an integrable F_1 -measurable random variable.

$$= \sum_{r<s\le t} \frac{E_s(X)X_s}{1-E_s(X)\,1-E_s(X)+E_s(X)X_s} \int_r^t \frac{E_s(X)}{(1-E_s(X))^2}ds =$$

$$= \int_r^t \int_{R_+} \frac{E_s(X)x}{1-E_s(X)\,1-E_s(X)+E_s(X)x}(dxds) \int_r^t \frac{E_s(X)}{(1-E_s(X))^2}$$

$$= \int_r^t \int_{R_+} \frac{H(s;x)}{1-E_s(X)}(ds;dx) \int_r^t \frac{E_s(X)}{(1-E_s(X))^2}ds: \qquad (16)$$

By the Itô formula and (16) for any $r > 0$

$$\frac{L_t}{1-E_t(X)} - \frac{L_r}{1-E_r(X)} = \qquad (17)$$

$$= \int_r^t \frac{1}{1-E_s(X)}dA_s + \int_r^t \frac{1}{1-E_s(X)}dM_s +$$

$$+ \int_r^t L_s\, d(1-E_s(X))^{-1}) + [L;(1-E(X))^{-1}]_t - [L;(1-E(X))^{-1}]_r =$$

$$= \int_r^t \frac{1}{1-E_s(X)}dA_s + \int_r^t \frac{1}{1-E_s(X)}dM_s - \int_r^t \frac{L_sE_s(X)}{(1-E_s$$

$$\int_r^t \int_{R_+} L_s \frac{H(s;x)}{1-E_s(X)}(ds;dx) - \int_r^t \int_{R_+} K(s;x)\frac{H(s;x)}{1.1-E_s(X)}$$

$$\int_r^t X$$

s

Since by (13) the left-hand side of (18) is a martingale on the interval $[r; 1]$ for any $r > 0$, the bounded variation part in (18) should be equal to zero. Therefore,

$$A_t \quad A_r = \int_r^t L_s \frac{E_s(X \quad)}{1}$$

By the boundary condition (12) and the Levy theorem, passing to the limit as $t \to 1$ in (22) we obtain that

$$= Y_1 = L_1 ;$$

which by arbitrariness of $r > 0$ and right-continuity of Y and M_t implies that $Y_t = L_t = E(L_1 \, jF_t)[1 \quad E_t(X \quad)]$ for any $t \quad 0$.

Remark 1. If X

Since $0 < X_t < 1; t$

and substituting this expression in (20), we obtain that

$$A_t = \int_0^t \int_{\mathbb{R}_+} L_s \frac{H(s;x)}{1 - H(s;x)}\,(dx)ds + \int_0^t L_s \frac{E_s(X)}{1 - E_s(X)}ds: \qquad (28)$$

By de nition of H

$$\frac{H(s;x)}{1 - H(s;x)} = \frac{xE(X)}{1 - E_s(X)};$$

)hj /T1_0impliesj /T1_0that7(28).

therefore

$$A_t = \int_0^t \int_{\mathbb{R}_+} L_s \; xE_s(X)X \;).794\ \text{Td i341.955 Tf 24.506 0 Td ())Tj4(0)Tj /T1\_1 11.95}$$

Stochastic Volatility Model with Small Randomness. Construction of CULAN Estimators

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2222

where $w = (w^R; w)$ is a standard two-dimensional Wiener process, defined on complete probability space $(\ ; F; P)$, $F^w = (F_t^w)_{0 \le t \le T}$ is the P-augmentation of the natural filtration $F_t^w = \sigma(w_s;\ 0 \le s \le t)$, $0 \le t \le T$, generated by w, $f(\ )$ is a continuous one-to-one positive locally bounded function (e.g., $f(x) = e^x$), $\ = (\ _1; : : : ;\ _m)$, $m \ge 1$, is a vector of unknown parameters, and ε, $0 < \varepsilon < 1$, is a small number. Assume that the system (1.1) has an unique strong solution.

Suppose that the sample path $y^{(\ )} = (y_s^{(\ )})_{0 \le s \le t}$ comes from the observations of process $Y^{\varepsilon,\ } = (Y_s^{\varepsilon,\ })_{0 \le s \le t}$ with distribution $P^{\varepsilon,\ }$ from the shrinking contamination neighborhood of the distribution $P^{\ }$ of the basic process $Y = (Y_s)_{0 \le s \le t}$. That is,

$$\frac{dP^{\varepsilon,\ }}{dP^{\ }} \mid F_t^w = \mathcal{E}_t(\varepsilon N^{\ }); \tag{1.2}$$

where $N^{\ } = (N_s^{\ })_{0 \le s \le t}$ is a $P^{\ }$-square integrable martingale, $\mathcal{E}_t(M)$ is the Dolean exponential of martingale M.

In the diffusion-type processes framework (1.2) represents the Huber gross error model (as it explain in Remark 2.3). The model of type (1.2) of contamination of measures for statistical models with filtration was suggested

2 Construction of CULAN estimators

2.1 Basic model

The basic model of observations is described by the SDE

$$dY_s = a(s, Y; \theta)\,ds + \varepsilon\,dw_s, \quad Y_0 = 0, \quad 0 \le s \le t, \tag{2.1}$$

where t is a fixed number, $w = (w_s)_{0 \le s \le t}$ is a standard Wiener process defined on the filtered probability space $(\Omega, \mathcal{F}, \mathbf{F} = (\mathcal{F}_s)_{0 \le s \le t}, P)$ satisfying the usual conditions, $\theta = (\theta_1, \ldots, \theta_m)$, $m \ge 1$, is an unknown parameter to be estimated, $\theta \in A \subset R^m$, A is an open subset of R^m, ε, $0 < \varepsilon \le 1$, is small parameter (index of series). In our further considerations all limits correspond to $\varepsilon \to 0$.

Denote by $(C_t, \mathcal{B}_t)$ a measurable space of continuous on $[0, t]$ functions $x = (x_s)_{0 \le s \le t}$ with σ-algebra $\mathcal{B}_t = \mathcal{B}(x : x_s,\ s \le t)$. Put $\mathcal{B}_s = \mathcal{B}(x : x_u,\ u \le s)$.

Assume that for each $\theta \in A$, the drift coefficient $a(s, x; \theta)$, $0 \le s \le t$, $x \in C_t$ is a known nonanticipative (i.e., $\mathcal{B}_s$-measurable for each s, $0 \le s \le t$) functional satisfying the functional Lipshitz and linear growth conditions L:

$$|a(s, x^1; \theta) - a(s, x^2; \theta)| \le L_1 \int_0^s |x^1_u - x^2_u|\,dk_u + L_2 |x^1_s - x^2_s|;$$

$$|a(s, x; \theta)| \le L_1 \int_0^s (1 + |x_u|)\,dk_u + L_2 (1 + |x_s|);$$

where L_1 and L_2 are constants, which do not depend on θ, $k = (k(s))_{0 \le s \le t}$ is a non-decreasing right-continuous function, $0 \le k(s) \le k_0$, $0 < k_0 < 1$, $x^1, x^2 \in C_t$.

Then, as it is well known (see, e.g., Lipster and Shiryaev [2]), for each $\theta \in A$, the equation (2.1) has an unique strong solution $Y^\varepsilon(\theta) = (Y^\varepsilon_s(\theta))_{0 \le s \le t}$ and, in addition (see Kutoyants [3]),

$$\sup_{0 \le s \le t} |Y^\varepsilon_s(\theta) - Y^0_s(\theta)| \le C\varepsilon \sup_{0 \le s \le t} |w_s| \quad P\text{-a.s.};$$

with some constant $C = C(L_1, L_2, k_0, t)$, where $Y^0(\theta) = (Y^0_s(\theta))_{0 \le s \le t}$ is the solution of the following nonperturbated differential equation

$$dY_s = a(s, Y; \theta)\,ds, \quad Y_0 = 0. \tag{2.2}$$

Change of initial problem of estimation of parameter by the equivalent one, when the observations are modelled according to the following SDE

$$dX_s = a_{"}(s; X;) ds + dw_s; \quad X_0 = 0; \tag{2.3}$$

where $a_{"}(s; x;) = \frac{1}{"} a(s; "x;), 0 \quad s \quad t, x 2 C_t, \quad 2 A .$

It it clear that if $X^{"}( ) = (X^{"}_s( ))_{0 s t}$ X ( ))

is well-de ned, and moreover, uniformly in on each compact,

$$P^{"} \quad \lim_{"!0} "^2 I_t^{"}(\ ) = I_t^0(\ ); \tag{2.7}$$

where

$$I_t^0(\ ) := \int_0^t \underline{a}(s; Y^0(\ ); \ )[\underline{a}(s; Y^0(\ ); \ )]^0 ds:$$

For each 2 , introduce the functional $_{"}(s; x; \ ) := \frac{1}{"} \ (s; "x; \ )$ and matrices $_{t"}(\ )$ and $_{t"}(\ )$:

$$_{t"}(X; \ ) := \int_0^t \ _{"}(s; X; \ )[\ _{"}(s; X; \ )]^0 ds; \tag{2.8}$$

$$_{t"}(X; \ ) := \int_0^t \ _{"}(s; X; \ )[\underline{a}_{"}(s; X; \ )]^0 ds: \tag{2.9}$$

Then from (2.6) it follows that uniformly in on each compact,

$$P^{"} \quad \lim_{"! \ 0} "^2 \ _{t"}(\ ) = \ _{t0}(\ ); \tag{2.10}$$

$$P^{"} \quad \lim_{"! \ 0} "^2 \ _{t"}(\ ) = \ _{t0}(\ ); \tag{2.11}$$

where the matrices $_{t0}(\ )$ and $_{t0}(\ )$ are de ned as follows:

$$_{t0}(\ ) = \int_0^t \ (s; Y^0(\ ); \ )[\ (s; Y^0(\ ); \ )]^0 ds; \tag{2.12}$$

$$_{t0}(\ ) = \int_0^t \ (s; Y^0(\ ); \ )[\underline{a}(s; Y^0(\ ); \ )]^0 ds: \tag{2.13}$$

Note that, by virtue of (2.4), (2.5) and $\underline{a}$ 2 , matrices given by (2.8), (2.9), (2.12) and (2.13) are well de ned.

Denote by $_0$ the subset of such that for each 2 $_0$ and 2 A , rank $_{t0}(\ )$ = m and rank $_{t0}(\ )$ = m.

Assume that $\underline{a}$ 2 $_0$.

For each 2 $_0$, de ne a $P^{"}$-square integrable martingale $L_t^{;"}(\ )$ as follows:

$$L_t^{;"}(X; \ ) := \int_0^t \ _{"}(u; X; \ )(dX_u \quad a_{"}(u; X; \ )du): \tag{2.14}$$

Now we give a de nition of CULAN

De nition 2.1. An estimator $(\ _t^{;"})_{"\to 0} = (\ _{1;t}^{;"}; : : : ; \ _{m;t}^{;"})_{"\to 0}^{0}$, $\ 2\ _0$, is called consistent uniformly linear asymptotically normal (CULAN) if it admits the following expansion:

$$\ _t^{;"} = \ + [\ _{t0}(\ )]^{1}\ "^2 L_t^{;"}(\ ) + r_{t"}(\ ); \tag{2.15}$$

where uniformly in $\$ on each compact,

$$P^{"}\ \lim_{"!\ 0} "^{1}\ r_{t"}(\ ) = 0: \tag{2.16}$$

It is well known (see Kutoyants [3]) that under the above conditions, uniformly in $\$ on each compact,

$$Lf"^{\ 1}(\ _t^{;"}\ \ \ ) \ j\ P^{"}g \ !^{w}\ N(0; V_t(\ \ ;\ \ ));$$

with

$$V_t(\ \ ;\ \ ) := [\ _{t0}(\ )]^{1}\ \ _{t0}(\ )([\ _{t0}(\ )]^{1})^{0}; \tag{2.17}$$

where $L(\ \ j\ P)$ denotes the distribution of random vector $\$, calculated under measure P, symbol $\!^{w}$ denotes the weak convergence of measures, $N(0; V_t(\ \ ;\ \ ))$ is a distribution of Gaussian vector with zero mean and covariance matrix $V_t(\ \ ;\ \ )$.

Remark 2.1. In context of di usion type processes, the M-estimator $(\ _t^{;"})_{"\to 0}$ is de ned as a solution of the following stochastic equation:

$$L_t^{;"}(X;\ \ ) = 0;$$

where $L_t^{;"}(X;\ \ )$ is de ned by (2.14), $\ 2\ _0$.

The asymptotic theory of M-estimators for general statistical models with ltration is developed in Toronjadze [4]. Namely, the problem of existence and global asymptotic behaviour of solutions is studied. In particular, the conditions of regularity and ergodicity type are established under which M-estimators have a CULAN property.

For our model, in case when $A = R^m$, the su cient conditions for CULAN property take the form:

(1) for all s, $0 \ s \ t$, and $x\ 2\ C_t$, the functionals $\ (s; x;\ \ )$ and $\underline{a}(s; x;\ \ )$ are twice continuously di erentiable in $\$ with bounded derivatives satisfying the functional Lipshitz conditions with constants, which do not depend on $\$.

(2) the equation (w.r.t. y)

$$\Psi_t(\lambda; y) := \int_0^t \psi(s, Y^0(\lambda); y)(a(s, Y^0(\lambda); \alpha) - a(s, Y^0(\lambda); y))\, ds = 0$$

has a unique solution $y = \alpha$.

The MLE is a special case of M-estimators when $\psi = \underline{a}$.

Remark 2.2. According to (2.7), the asymptotic covariance matrix of MLE $(\hat{\alpha}_t^\varepsilon)_{\varepsilon>0}$ is $[I_t^0(\lambda)]^{-1}$. By the usual technique one can show that for each $\alpha \in A$ and $\lambda \in \Lambda_0$, $[I_t^0(\lambda)]^{-1} \geq V_t(\alpha; \lambda)$, see (2.17), where for two symmetric matrices B and C the relation $B \geq C$ means that the matrix $C - B$ is nonnegative de nite.

Thus, the MLE has a minimal covariance matrix among all M-estimators.

2.2 Shrinking contamination neighborhoods

In this subsection, we give a notion of a contamination of the basic model (2.3), described in terms of shrinking neighborhoods of basic measures P_α^ε, $\alpha \in A; \varepsilon > 0\}$, which is an analog of the Huber gross error model (see, e.g., Hampel et al. [5] and, also, Remark 2.3 below).

Let H be a family of bounded nonanticipative functionals $h : [0, t] \times C_t \times A \to R^1$ such that for all $s \in [0, t]$ and $\alpha \in A$, the functional $h(s, x; \alpha)$ is continuous at the point $x_0 = Y^0(\lambda)$.

Let for each $h \in H$, $\alpha \in A$ and $\varepsilon > 0$, $P^{\varepsilon;h}$ be a measure on (C_t, B_t) such that

$$P^{\varepsilon;h} \geq (1 - \varepsilon) P_\alpha^\varepsilon$$

We call $(P^{\varepsilon;H})_{\varepsilon>0}$ a shrinking contamination neighborhoods of the basic measures $(P^{\varepsilon})_{\varepsilon>0}$, and the element $(P^{\varepsilon;h})_{\varepsilon>0}$ of these neighborhoods are called alternative measures (or, simply, alternative).

Obviously, for each $h \in H$ and $\alpha \in A$, the process $N^{\varepsilon;h} = (N^{\varepsilon;h}_s)_{0 \le s \le t}$ defined by (2.19) is a P^{ε}-square integrable martingale. Since under measure P^{ε} the process $\overline{w} = (\overline{w}_s)_{0 \le s \le t}$ defined as

$$\overline{w}_s := X_s - \int_0^s a_{\varepsilon}(u, X_{\cdot}(\alpha))\, du, \quad 0 \le s \le t,$$

is a Wiener process, by virtue of the Girsanov Theorem the process $w_s := \overline{w} + \langle \overline{w}, \varepsilon N^{\varepsilon;h} \rangle$ is a Wiener process under changed measure $P^{\varepsilon;h}$. But by the definition,

$$w_s = X_s - \int_0^s (a_{\varepsilon}(u, X_{\cdot}(\alpha)) + \varepsilon h_{\varepsilon}(u, X_{\cdot}(\alpha)))\, du,$$

and hence one can conclude that $P^{\varepsilon;h}$ is a weak solution of SDE

$$dX_s = (a_{\varepsilon}(s, X_{\cdot}(\alpha)) + \varepsilon h_{\varepsilon}(s, X_{\cdot}(\alpha)))ds + dw_s, \quad X_0 = 0.$$

)) (

Thus

$$\frac{dP^{n;h}}{dP^n} = E_n(\varepsilon_n \quad N^{n;n}) \tag{2.20}$$

and the relation (2.18) is a direct analog of (2.20).

2) The concept of shrinking contamination neighborhoods, expressed in the form of (2.18), was proposed in Lazrieva and Toronjadze [1] for more general situation, concerning with the contamination areas for semimartingale statistical models with ltration. □

In the remainder of this subsection, we study the asymptotic properties of CULAN estimators under alternatives.

For this aim, we rst consider the problem of contiguity of measures $(P^{\varepsilon;h})_{\varepsilon>0}$ to $(P^{\varepsilon})_{\varepsilon>0}$.

Let $(\varepsilon_n)_{n\ 1}$, $\varepsilon_n \# 0$, and $(\ _n)_{n\ 1}$, $\ _n \ 2 \ K$, $K \quad A$ is a compact, be arbitrary sequences.

Proposition 2.1. *For each* $h \ 2 \ H$, *the sequence of measures* $(P^{\varepsilon_n;h_n})$ *is contiguous to sequence of measures* $(P^{\varepsilon_n}_n)$, *i.e.,*

$$(P^{\varepsilon_n;h}_n) \ / \ (P^{\varepsilon_n}_n):$$

Proof. From the predictable criteria of contiguity (see, e.g., Jacod and Shiryaev [6]), it follows that we have to verify the relation

$$\lim_{N\ !1} \ \limsup_{n!1} P^{\varepsilon_n;h}_n \left\{ h^n_t \ \frac{1}{2} \ > N \right\} = 0; \tag{2.21}$$

where $h^n_t(\frac{1}{2}) = (h^{\ n}_s(\frac{1}{2}))_{o\ s\ t}$ is the Hellinger process of order $\frac{1}{2}$.

By the de nition of Hellinger process (see, e.g., Jacod and Shiryaev [6]), we have

$$h^n_t \ \frac{1}{2} \ = h^n_t \ \frac{1}{2}; P^{\varepsilon_n;h}_n; P^{\varepsilon_n}_n \ = \frac{1}{8} \int_0^t [h(s; \varepsilon_n X; \ _n)]^2 ds;$$

and since $h \ 2 \ H$, and hence is bounded, $h^n_t(\frac{1}{2})$ is bounded too, which provides (2.21). □

Proposition 2.2. *For each estimator* $(\ ^{\varepsilon;}_t)_{\varepsilon>0}$ *with* $2 \quad _0$ *and each alternative* $(P^{\varepsilon;h})_{\varepsilon>0} \ 2 \ (P^{\varepsilon;h})_{\varepsilon>0}$, *the following relation holds true:*

$$Lf\varepsilon^{\ 1}(\ ^{\varepsilon;\varepsilon}_t \quad) \ j \ P^{\varepsilon;h}g \ !^{w} \ N([\ _{t0}(\ )]^{\ 1} \ b_t(\ ; h \ ; \ ); V_t(\ ; \ ));$$

where

$$b_t(\ ; h; \ ) := \int_0^t (s; Y^0(\ ); \ )h(s; Y^0(\ ); \ ) \, ds:$$

Proof. Proposition 2.1 together with (2.16) provides that uniformly in on each compact,

$$P^{";h} \lim_{"!0} "^{1} r_{t"}( ) = 0 ;$$

and therefore we have to establish the limit distribution of random vector $[_{t0}( )]^{1} "L_t^{;"}$ under the measures $(P^{"h})_{">0}$.

By virtue of the Girsanov Theorem, the process $L_t^{;"}( ) = (L_s^{;"}( ))_{0 s t}$ is a semimartingale with canonical decomposition

$$L_s^{;"}( ) = e_s^{;"}( ) + b_{";s}(; h ;); \quad 0 \quad s \quad t; \qquad (2.22)$$

where $e_t^{;"}( ) = (e_s^{;"}( ))_{0 s t}$ is a $P^{";h}$-square integrable martingale, de ned as follows:

$$e_s^{;"}(X ;) := \int_0^s {}_{"}(u; X ;)(dX_u \quad (a_{"}(u; X ;) + "h_{"}(u; X ;))du);$$

and

$$b_{";s}(; 4t( )22=9.8Td( ))$$
$$\int_0^{\quad} (u; X ; \quad (u; X ; \quad (\quad ($$

Initially, it should be stressed that the bias vectors θ

Therefore

$$D_t(\ ;h;\quad) = D_t(\overset{e}{\ };h;\quad) = jb_t(\overset{e}{\ };h;\quad)j^2 + tr\ \overset{e}{}_{t0}(\ ): \qquad (2.24)$$

Denote by H_r a set of functions$h \in H$ such that for each $\in A$,

$$\int_0^t jh(x; Y^0(\ );\ )j\,ds \quad r;$$

where$r$, $r > 0$, is a constant.

Since, for each$r > 0$,

$$\sup_{h \in H_r} jb_t(\overset{e}{\ };h;\quad)j \quad const.(r) \sup_{0 \le s \le t} j\ \overset{e}{}_t(s; Y^0(\ );\ )j;$$

where constant depends on, we call the function e an in uence function of estimator $(\ _t^{;"})_{"\to 0}$ and a quantity

$$_t\ (\ ) = \sup_{0 \le s \le t} j\ \overset{e}{}(s; Y^0(\ );\ )j$$

is named as the (unstandardized) gross error sensitivity at point of estimator $(\ _t^{;"})_{"\to 0}$.

De ne

$$_{0;c} = \qquad\qquad r, \text{ we call is} \qquad \text{call} j\ ;c$$

Theorem 2.1. Assume that for given constant c there exists a nondegenerate $m \times m$ matrix $A_c(\cdot)$, which solves the equation (w.r.t. matrix A)

$$\int_0^t {}^{A}_{c}(s; Y^0(\cdot); \cdot)[\underline{a}(s; Y^0(\cdot))]^0 ds = Id. \tag{2.28}$$

Then the function ${}^{A_c(\cdot)}_{c} = h_c(A_c(\cdot)\underline{a})$ solves the optimization problem (2.27).

Proof. (See, e.g., Hampel et al. [5].)

Let A be an arbitrary $m \times m$ matrix.

Since for each $\in {}_0$, $(\underline{a})^0 = Id$, $\int \underline{a}[\underline{a}]^0 = I^0(\cdot)$ (see (2.7)) and the trace is an additive functional, we have

$$(A\underline{a})(A\underline{a})^0 = \int {}^0 A A^0 + AI^0(\cdot)A^0$$

(here and below we use simple evident notations for integrals).

Therefore instead of minimizing of $\mathrm{tr} \int {}^0$ we can minimize

$$\mathrm{tr} \int (A\underline{a})(A\underline{a})^0 = \int |A\underline{a}|^2;$$

and it is evident that a function $h_c(A\underline{a})$ minimizes the expression under integral sign, and hence the integral itself over all functions $\in {}_0$, satisfying (2.26).

At the same time, the condition (2.25), generally speaking, can be violated. But, since a matrix A is arbitrary, we can choose $A = A_c(\cdot)$ from (2.28) which guarantees the validity of (2.25) with ${}_c = {}^{A_c(\cdot)}_{c}$. □

As we have seen, the resulting optimal in uence function ${}_c$ is de ned along the process $Y^0(\cdot) = (Y^0_s(\cdot))_{0 \le s \le t}$, which is a solution of equation (2.2).

But for constructing optimal estimator we need a function ${}_c(s; x; \cdot)$, de ned on whole space $[0; t] \times C_t \times A$.

For this purpose, de ne ${}_c(s; x; \cdot)$ as follows:

$${}_c(s; x; \cdot) = {}^{A_c(\cdot)}_{c}(s; x; \cdot) = h_c(A_c(\cdot)\_t \quad {}_c(\cdot)$$

From (2.9), (2.11), (2.28) and (2.29) it directly follows that

$$_{t0}^{c}(\ ) = P^{"} \lim_{"!0} "^{2}\ _{t"}^{c}(\ ) = \int_{0}^{t} {}_{c}(s; Y^{0}(\ );\ )(\underline{a}(s; Y^{0}(\ );\ ))^{0}ds = Id:$$

Besides, for each alternative $(P^{";h})_{">0}$, $h \in H$, according to Proposition 2.2, we have

$$Lf"^{-1}(\ _{t}^{;"}\ ) \mid P^{";h}g \xrightarrow{w} N(b_t(\ _{c}; h;\ ); V_t(\ _{c};\ ))\ \text{ as }\ " \to 0;$$

where

$$b_t(\ _{c}; h;\ ) = \int_{0}^{t} {}_{c}(s; Y^{0}(\ );\ )h(s; Y^{0}(\ );\ )ds;$$

and $V_t(\ _{c};\ ) = \ _{t0}^{c}(\ )$.

Hence, the risk functional for estimator $(\ _{t}^{;"})_{">0}$ is

$$D_t(\ _{c}; h;\ ) = |b_t(\ _{c}; h;\ )|^2 + tr\ _{t0}^{c};\quad h \in H;$$

and the (unstandardized) gross error sensitivity of $(_{t}^{;"})_{">0}$ is

$$_{c}(\ ) = \sup_{0 \le s \le t} |\ _{c}(s; Y^{0}(\ );\ )| \quad c:$$

Thus, we may conclude that $(\ _{t}^{;"})_{">0}$ is the optimal B-robust estimator over the class of estimators $(_{t}^{;"})_{">0}$, $\in\ _{0;c}$, in the following sense: the trace of asymptotic covariance matrix of $(\ _{t}^{;"})_{">0}$ is minimal among all estimators $(\ _{t}^{;"})_{">0}$ with bounded by constant c gross error sensitivity, that is,

$$_{t0}^{c}(\ ) \quad _{t0}(\ ) \ \text{ for all }\ \in\ _{0;c}:$$

Note that for each estimator $(\ _{t}^{;"})_{">0}$ and alternatives $(P^{";h})_{">0}$, $h \in H$, the influence functional is bounded by $const(r)\ c$. Indeed, we have for $\in\ _{0;c}$,

$$\sup_{h \in H_r} |b_t(\ ; h;\ )| \quad const(r)\ \ c := C(r; c);$$

and since from (2.24)

$$\inf_{\in\ _{0;c}} \sup_{h \in H} {}_{t}(\ ; h;\ ) = \ ; c$$

over all constants c, for which the equation (2.28) has a solution $A_c(\ )$. This can be done using the numerical methods.

For the problem of existence and uniqueness of solution of equation (2.28), we address to Rieder [7].

In the case of one-dimensional parameter (i.e., $m = 1$), the optimal level c of truncation is given as a unique solution of the following equation (see Lazrieva and Toronjadze [1])

$$r^2c^2 = \int_0^{Z_t} [\underline{a}(s; Y^0(\ ); \ )]_c^c \ \underline{a}(s; Y^0(\ ); \ )ds \int_0^{Z_t} ([\underline{a}(s; Y^0(\ ); \ )]_c^c)^2 ds;$$

where $[x]_a^b = (x \wedge b) \_ a$ and the resulting function

$$(s; x; \ ) = [\underline{a}(s; x; \ )]_c^c; \quad 0 \quad s \quad t; \quad x \ 2 \ C_t;$$

is $(\ _0; H_r)$ optimal in the following minimax sense:

$$\sup_{h2H_r} D_t(\ \ ; h; \ ) = \inf_2 \sup_{h2H_r} D_t(\ ; h; \ \ ):$$

Appendix

Important feature of the stochastic volatility model is that volatility process Y is unobservable (latent) process. Clear that full knowledge of the model of the process Y is necessary and hence one needs to estimate the unknown parameter $= (\ _1; : : : ; \ _m)$, $m \quad 1$.

A variety of estimation procedures are used, which involve either direct statistical analysis of the historical data or the use of implied volatilities extracted from prices of existing traded derivatives.

Consider the method based on historical data.

Fix the time variable t. From observations $Y_{t_0^{(n)}}; : : : ; Y_{t_n^{(n)}}$, $0 = t_0^{(n)} < < t_n^{(n)} = t$, $\max_j [t_{j+1}^{(n)} \quad t_j^{(n)}] \ ! \quad 0$ as $n \ ! \quad 0$, calculate the realization of yield process $R_t = \int_0^t \frac{dY_s}{Y_s}$, and then calculate the sum

$$S_n(t) = \sum_{j=0}^{n-1} R_{t_{j+1}^{(n)}} \quad R_{t_j^{(n)}} \ ^2:$$

It is well known (see, e.g., Lipster and Shiryaev [2]) that

$$S_n(t) \ !^P \int_0^{Z_t} \ ^2_s ds \quad \text{as } n \ ! \ 1:$$

Since $\sigma_t^2(\omega) = f(Y_t)$ is a continuous process, we get

$$\sigma_t^2(\omega) = \lim_{\Delta \to 0} \frac{F(t+\Delta;\omega) - F(t;\omega)}{\Delta};$$

where $F(t;\omega) = \int_0^t \sigma_s^2(\omega)\, ds$.

Hence, the realization $(y_t)_{0 \le t \le T}$ of the process Y can be found by the formula $y_t = f^{-1}(\sigma_t^2)$, $0 \le t \le T$.

We can use the reconstructed sample path $(y_t(\omega))$, $0 \le t \le T$, to estimate the unknown parameter θ in the drift coefficient of diffusion process Y.

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EDTECH SOLUTIONS FOR TRAINING ENGAGEMENT IN CLIP CORPORATE CULTURE

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Abstract

Nowadays organizations are facing challenges which cannot be resolved by the traditional managerial solutions. EdTech area has a relatively short history, it is a sphere in dynamic development process. Learning and Development practices in organizations tend to integrate and use the novelties from different training platforms. It stared with Excel, E-mails and Google Docs and today is present by complex solutions in Training Management Systems. The dynamism, uncertainty and high competition in business organizations make them adapt their learning systems and corporate cultures. From the comparative values organizational culture framework of Robert E. Quinn and Kim S. Cameron [1] companies are moving to 'clip' corporate culture and need of a different AI-based and scenario-based training system approaches.

Key words:

More service style by leaders

Human resources development urgency (as reskilling, upskilling, etc.)

Scaling of the learning programs (EdTech development)

HR Branding and talent management (retaining talented employees with the help of positive organization's image)

Moreover, in organizations as reflecting overall culture development trends, there is a tendency to simulation of reality to become reality (simulacra and simulation of Jean Baudrillard[3]). In this system, the symbolic leadership and generally symbolic meanings play a greater role in managerial effectiveness. It is not enough to plan, organize, lead and control. It is a demand for managing intangible elements in communication, team work, relationships and meanings at work (purpose, mission, goals, vision, adjusted to employees needs, or at least in the average fit).

Corporate University for Scaling Learning and Development

In the light of the new meanings and directions, Corporate University takes a greater space in corporate Learning and Development programs (see Exhibit 1). Employee Learning usually considers newly hired employees (to train for the job and position and to teach standards) and current employees to keep them up-to-date (competences development; changes in conditions; changes of specific or general environment; company changes; talent pool/talent management). Moreover, the whole L&D System is set to promote high performance (Exhibit 2). The skills are divided into two categories: hard and soft skills (Exhibit 3), which means that the approach to Learning and Development and the selected solution should be useful for developing both clusters of skills. They have some specificity and need variable m

Exhibit 1

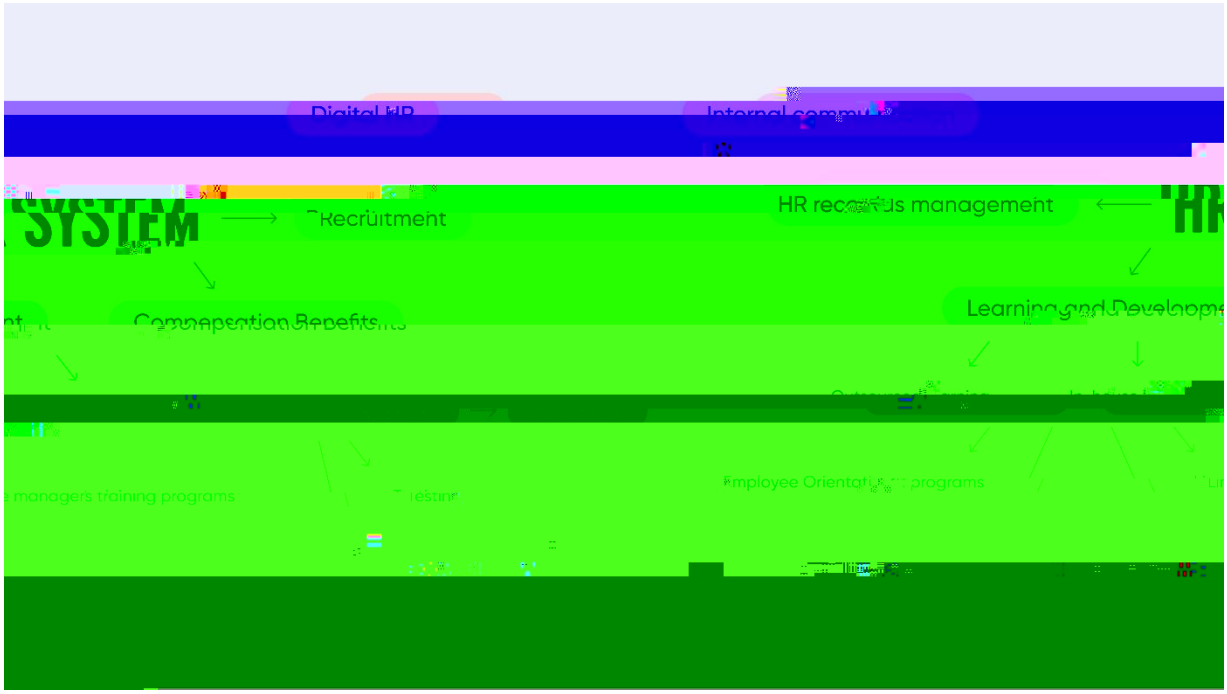


Exhibit 2

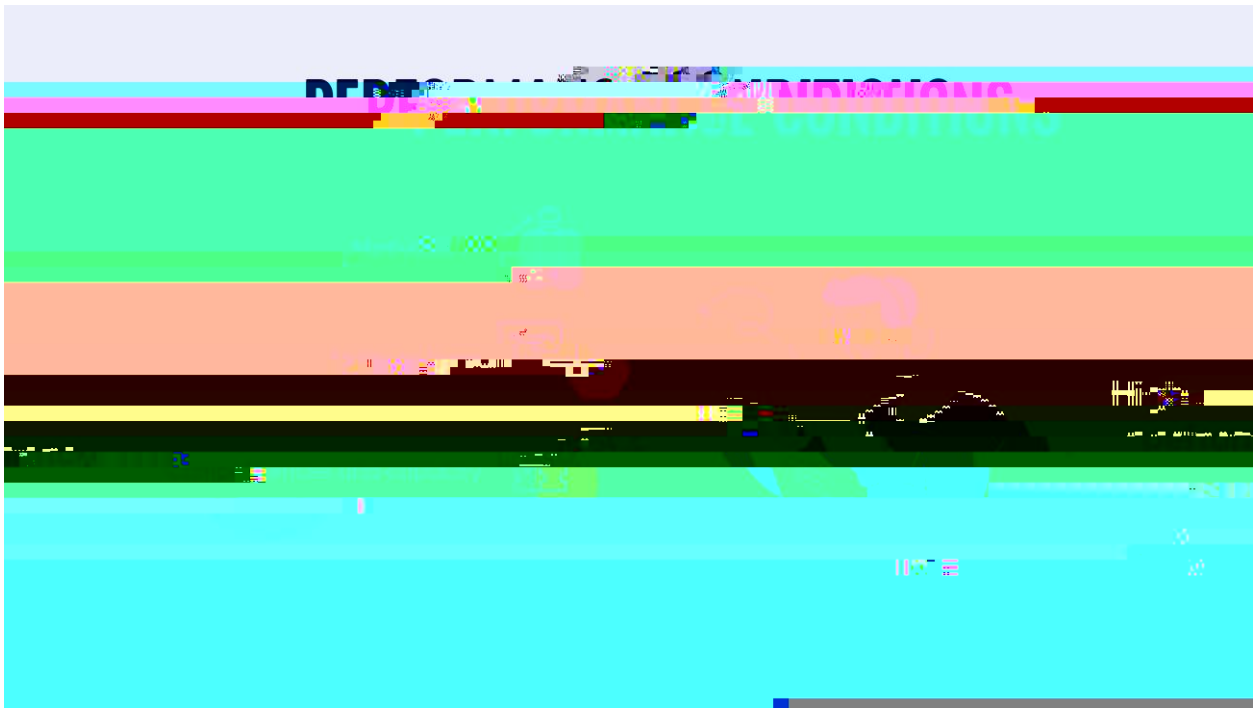


Exhibit 3

Corporate University resolved this demand and implements it in the complex strategic approach to corporate learning and development, as well as s pproave strategic

Exhibit 4

EdTech Market Today

EdTech is the practice of introducing information and communication technology tools into the classroom to create more engaging, inclusive and more individualized learning experience; hardware and software to enhance the quality of learning experience and get to high performance results. EdTech market has been permanently developing, as the demand is high. Nowadays it is present by the following services (Exhibit 5):

LMS (Learning Management Systems) for asynchronous learning (e.g. Moodle)

Online whiteboards (e.g. Miro and Mural)

Engagement tools (for quizzes, polls, etc., as e.g. Mentimeter, Kahoot)

E-learning (E-learning courses, as e.g. Gurucan, CourseLab)

LXP (Learning Experience Platform)

Video Conferencing and Webinars (as Zoom, Webex, MsTeams)

VA and AR (Artificial Intelligence systems)

TMS (Training Management Systems) for complex solutions (blended learning), (as Lanes, AdobeConnect)

Training Management Systems are developing the instruments and mechanics integrated in one platform to enhance the engagement and allow scenario training scaling. The main challenges for virtual learning and development are:

- Team work clarity and standards
- Scaling
- Engagement
- Measuring learning outcomes
- Optimizing the work of L&D Team
- Adapting the content for virtual learning
- Balancing workload
- Continuous improvement
- Feedback and assessment

The results with Training Management System can be impressive. For example, with Lanes platform [4] (some cases examples).

In Banking: Training sessions reduced to 3 hours, scalability enhanced, life-long learning culture and retention, new program design reduced to 1-2 days (compared to 2 weeks).

Healthcare company: cost training per employee reduced from $700 to $7, expertise developed company-wide easily, improved engagement and learning results.

Fast-food chain: 23% higher engagement rate, waiting period for leadership dramatically cut (from 6 months down to 3 days), expenditure on company-wide mandatory training has decreased by 39%.

Nowadays the leaders among TMS platforms worldwide are AdobeConnect[5] and Lanes[6] Exhibits 8-10).

Exhibit 8 AdobeConnect

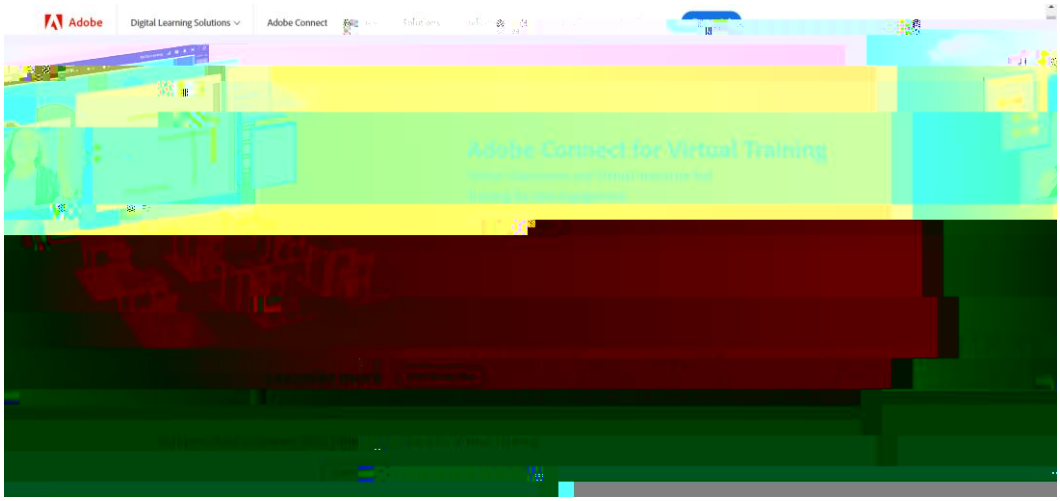
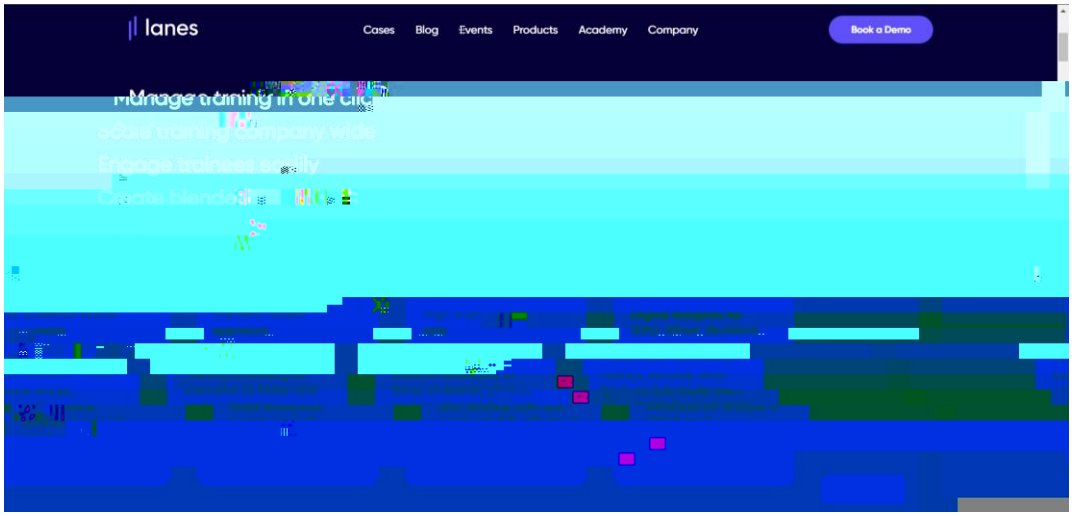


Exhibit 9 Lanes

Exhibit 10 Lanes



2. Main Result

Investigating the Subgaussian random elements with values in Banach spaces and analyzing the results of [2], R. Fukuda [3] came to a result, which is improved in our Theorem 2 stated below.

Let $(\ ;\ A;P)$ be a probability space, $:\ \ !\ \ R^1$ be a real random variable and E be a mathematical expectation symbol.

Theorem 2. Let $p > q > 0$ and for some $C\ \ 1$

$$fEj\ \ j^p g^{1=p}\ \ C\, fEj\ \ j^q g^{1=q} < 1: \tag{1}$$

Then for any $r;s,\ 0 < r;s\ \ p$, we have

$$fEj\ \ j^r g^{1=r}\ \ C\ \ fEj\ \ j^s g^{1=s};$$

where

$$= \begin{cases} 0; & \text{if } 0 < r\ \ s\ \ p; \\ 1; & \text{if } q\ \ s < rjq \end{cases}$$

Now the case $0 < s < r \quad q$ is left, which can be reduced to the previous one.

Note that applying Kahane's inequality, Fukuda in his paper [3] as a constant C for $r = p$ and $s = 1$ obtained the expression

$$C^{1+\frac{pq}{p\ q}} \quad q \quad B^{\ 1}\ (1=q; \frac{p}{p \quad q} + 1); \tag{4}$$

where $B(\ ;\ )$ is a beta function.

For the same values of the parameters $(\neq p; s = 1)$, the constant obtained from Theorem 2 is equal to

$$C \ = \ \begin{cases} C; & \text{if} \quad 0 < q \quad 1; \\ C^{\frac{q(p\ 1)}{p\ q}}; & \text{if} \quad 1 < q < p: \end{cases} \tag{5}$$

Using the computer program MAPLE we compared the values of (4) and (5) for di erent values of the parameters p and q, and it was found that the constant obtained by Theorem 2 is better, although it needs analytical con rmation.

Acknowledgment

The Impact of Promotions on Consumer Behavior

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ABSTRACT

In a high velocity environment retail market is growing all over the world. Retailers use all types of promotional activities in order to be differentiated in the market. As a result of population and economic growth, retailers started to widen the application of various marketing in order to influence consumers. Peattie and Peattie (1994) stated that marketing activities are usually specific to a time period, place or customer group, which encourage a direct response from consumers or marketing intermediaries, through the offer of additional benefits. One of these activities is to use promotions (such as discounts, buy one get one free, coupons, rebates, contests, cash-back offers and loyalty programs) and they directly influence individual to give quick decision and to finalize purchasing process. Since promotions are one of the most noticed marketing activities, promotions can greatly impact any company's market share and sustainability. It is therefore important to understand which promotions consumers prefer and also the effect of promotions on customers and their behavior. (Peattie, 1994)

According to the American Marketing Association, as noted by Kotler and Keller, marketing can E H G H I L Q H G D V ³D Q R U J D Q L] D W L R Q D O I X Q F W L R Q D O D Q G D V delivering value toconsud O I X Q Fe,also6 (a)3 (b)1-9 [(d)8na<004F>6 <0003>-9 <00490058-9 (e

F R P S D Q \ ¶ V P H V V D J H D F U R V V W R W K H F R Q V X P H U 7 K H I R X U

promotion, public relation and direct marketing (Juneja, 2018)

x

- There are discounts when products are temporarily offered at a lower price. This form of promotion is used by organizations in order to increase sales and attract new customers.

- Systems that allow customers to obtain a refund of some of the purchase price is known as cash-back. When cash-back are offered immediately at the time of purchase, this is an instant rebate. An organization uses thi

After gathering information, it is evaluated against a consumer's wants, needs, preferences, and financial resources, which are available for purchase.

At the purchase stage, the consumer will make a purchasing decision. The ultimate decision may be based on factors such as availability or price.

At the post-purchase evaluation stage, the consumer will decide whether the purchase actually satisfies her needs and wants (Kotler, 2003)

The research, which is used in this study, is descriptive in its nature. It can be explained by particular situation, telling some sort of things or some sort of noticeable facts. Research that explains the present situation instead of interpreting and making judgments is descriptive research (Creswell, 1994). The core purpose of descriptive research is to establish the accurateness of developed hypothesis that reflect the present position. This kind of research gives knowledge about the current scenario and concentrate on past or present for an instance in a community quality of life or customer attitude toward any marketing activity (Kumar, 2005).

The research was conducted in Carrefour, with a Carrefour consumer. Various products are the core business of the supermarket Carrefour. Their product offer is based on a number of unchanging principles, which are a broad selection, the lowest prices, the highest quality and compliance with manufacturing conditions.

Brochures positioned in front of supermarkets are also very likely to cause sales. This is due to the fact that consumers like to check which products are on promotion at the time of their visit to the supermarket. Although consumers may have an intended shopping list while visiting the supermarket, the discovery of product under promotion always pushes towards spending money on discounted items.

In case of promotion, which enable consumers to save money, they are always eneyawoi money

Demonstrations are not very influential with regards to purchasing behavior but are more likely

A stochastic model of predator-prey population dynamics

T. Kutalia and R. Tevzadze

Abstract

Abstract. We present results of an analysis for a randomized three-dimensional predator-prey model representing the dynamics of wolf-deer interactions.

1 Introduction

We study a predator { prey stochastic model (initiated by R. Chitashvili) in discrete and continuous time. For this we define the transition probabilities and Markov chain realization by the random difference schemes as well as the systems of stochastic differential equations. We show that in case of scaling, the solution of the system approaches the solution of the equation of ordinary differential equation. We present the graphs of these solutions for the specific parameter set and different initial conditions. Graphs illustrate the equilibrium points which the system approaches in infinite time.

2 The discrete time Model

Let us consider two populations: deer and wolves and assume that deer divide into the group of strong and week ones. Denote by $x; y; z$ the number of strong deer, week deer and wolves respectively.

Let $(x;y;z);\ (x;y;z);\ (x;y;z);\ (x;y;z)\ 2\ N^3$ be intensity of transitions

$(x;y;z)\ !\ (x\ \ 1;y;z);(x;y;z)\ !\ (x;y\ \ 1;z);(x;y;z)\ !\ (x;y;z\ \ 1)$

and $=\ ^{+}+\ ;\ =$

where ε_n is i.i.d. with uniform distribution, define Markov chain with such transition probabilities. Indeed

$$P(X_{n+1} = X_n + 1; Y_{n+1} = Y_n; Z_{n+1} = Z_n | X_n; Y_n; Z_n) = P(X_{n+1} = X_n + 1 | X_n; Y_n; Z_n)$$
$$= P(I_{(\varepsilon_n < p^+(X_n;Y_n;Z_n))} \cdot I_{(p^+(X_n;Y_n;Z_n)) \le \varepsilon_n < p^+(X_n;Y_n;Z_n)+p^-(X_n;Y_n;Z_n))} = 1 | X_n; Y_n; Z_n)$$
$$= P(\varepsilon_n < p^-(X_n; Y_n; Z_n) | X_n; Y_n; Z_n) = p^-(X_n; Y_n; Z_n)$$

and at cetera.

Assume that the share of strong deer increases proportionally to fraction of strong deer pairs into all deer pairs, i.e. by intensity $(x + y)\frac{x^2}{(x+y)^2}$. Similarly for week deers we get intensity $(x + y)(1 - \frac{x^2}{(x+y)^2})$. The mortality of strong and week deer is defined as $d^s x + e^s xz$; $d^w y + e^w yz$ respectively. We define the rate of fecundity and mortality of wolfs as: $z + \gamma^0(x + y)$; $z + \gamma^0\frac{z^2}{x+y}$. Therefore

$$\lambda^+(x; y; z) = \frac{x^2}{x + y}; \quad \lambda^-(x; y; z) = (d^s x + e^s xz);$$
$$\mu^+(x; y; z) = \frac{(x + y)^2 - x^2}{x + y}; \quad \mu^-(x; y; z) = (d^w y + e^w yz);$$
$$\gamma^+(x; y; z) = z + \gamma^0(x + y); \quad \gamma^-(x; y; z) = z + \gamma^0\frac{z^2}{x + y}:$$

We take $e^s = d^s = 0{:}2$; $e^w = d^w = 0{:}5$; $\alpha = \beta$; $\gamma^0 = 1$; $\gamma^0 = 1$ and $(X_0; Y_0; Z_0) = (8; 8; 8)$. We graphically present the solution of (1) below

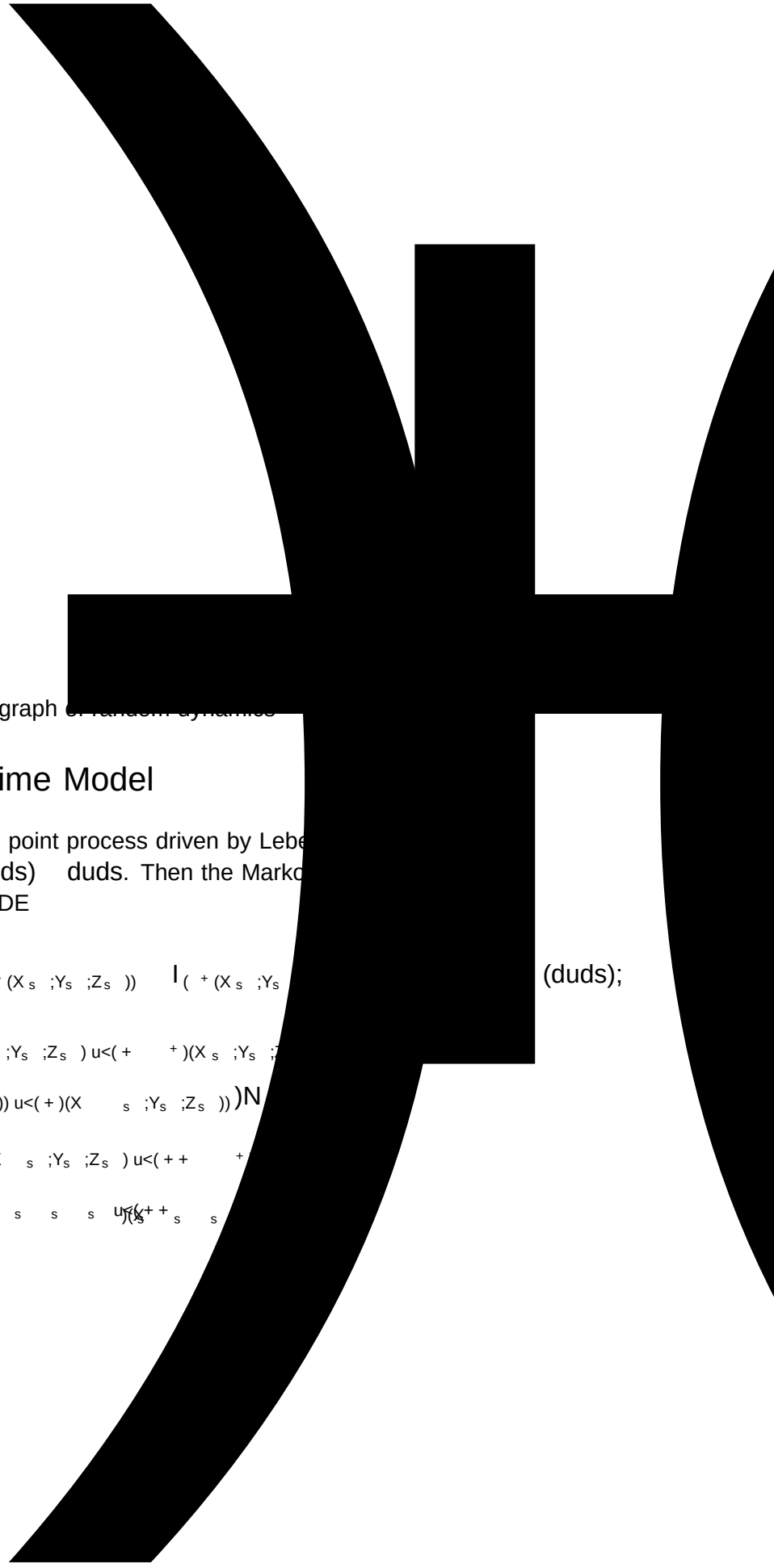

Figure 1: graph of random dynamics

3 The continuous time Model

Let $N(duds)$ be the Poisson point process driven by Lebe
$duds$ and $\tilde{N}(duds) = N(duds) \quad duds$. Then the Marko
uous time is defined by the SDE

$$X_t = X_0 + \int_0^t \int_0^1 (I_{(u< \quad ^+(X_s ;Y_s ;Z_s))} \quad I_{(\quad ^+(X_s ;Y_s} \quad (duds);$$

$$Y_t = Y_0 + \int_0^t \int_0^1 (I_{((X_s ;Y_s ;Z_s) u<(+ \quad ^+)(X_s ;Y_s ;}$$
$$I_{(+ \quad ^+)(X_s ;Y_s ;Z_s)) u<(+)(X \quad _s ;Y_s ;Z_s))})N$$

$$Z_t = Z_0 + \int_0^t \int_0^1 (I_{((+)(X \quad _s ;Y_s ;Z_s) u<(+ + \quad ^+}$$
$$I_{((+ + \quad _s \quad _s \quad _s \quad u<(+ + _s \quad _s \quad)(s}$$

Then

$$X_t = X_0 + \int_0^{Z_t} (\ ^{+}(X$$

In our model

$$^{+} = \frac{X^2}{X+Y}; \qquad = (d^s X + e^s XZ);$$

$$^{+} = \frac{(X+Y)^2 \quad X^2}{X+Y}; \qquad = (d^w Y + e^w YZ);$$

$$^{+} = Z + {}^{0}(X+Y); \qquad = Z + {}^{0}\frac{Z^2}{X+Y}:$$

Therefore

$$dX_t = \frac{X^2}{X+Y} \quad (d^s X + e^s XZ) \quad dt + dL(t)$$

$$dY_t = \frac{(X+Y)^2 \quad X^2}{X+Y} \quad (d^w Y + e^w YZ) \quad dt + dM(t)$$

$$dZ_t = \quad Z + {}^{0}(X+Y) \quad Z + {}^{0}\frac{Z^2}{X+Y} \quad dt + dN(t):$$

In deterministic case one obtains

$$\underline{x} = \frac{x^2}{x+y} \quad (d^s x + e^s xz);$$

$$\underline{y} = \frac{(x+y)^2 \quad x^2}{x+y} \quad (d^w y + e^w yz); \qquad (2)$$

$$\underline{z} = z + {}^{0}(x+y) \quad z \quad {}^{0}\frac{z^2}{x+y}:$$

Such type of population model was studied in [1].

Remark. If $\underline{y} = x + y$ then

$$\underline{x} = \frac{x^2}{\underline{y}} \quad x(d^s + e^s z);$$

$$\underline{y} = \underline{y}(1 \quad d^w \quad e^w z) + x(d^w + e^w z \quad d^s \quad e^s z);$$

$$\underline{z} = (\qquad)z + {}^{0}\underline{y} \quad {}^{0}\frac{z^2}{\underline{y}}:$$

In particular case of parameters we have

$$\begin{aligned} \underline{x} &= \frac{x^2}{x+y} \quad 0{:}2x(1+z); \\ \underline{y} &= \frac{(x+y)^2 \quad x^2}{x+y} \quad 0{:}5y(1+z); \\ \underline{z} &= (x+y) \quad \frac{z^2}{x+y}{:} \end{aligned} \tag{3}$$

or for $(x; \cancel{y}; z) = (x; x+y; z)$

$$\begin{aligned} \underline{x} &= \frac{x^2}{\cancel{y}} \quad 0{:}2x(1+z); \\ \cancel{y} &= \cancel{y} \quad (0{:}5\cancel{y} \quad 0{:}3x)(1+z); \\ \underline{z} &= \cancel{y} \quad \frac{z^2}{\cancel{y}}{:} \end{aligned}$$

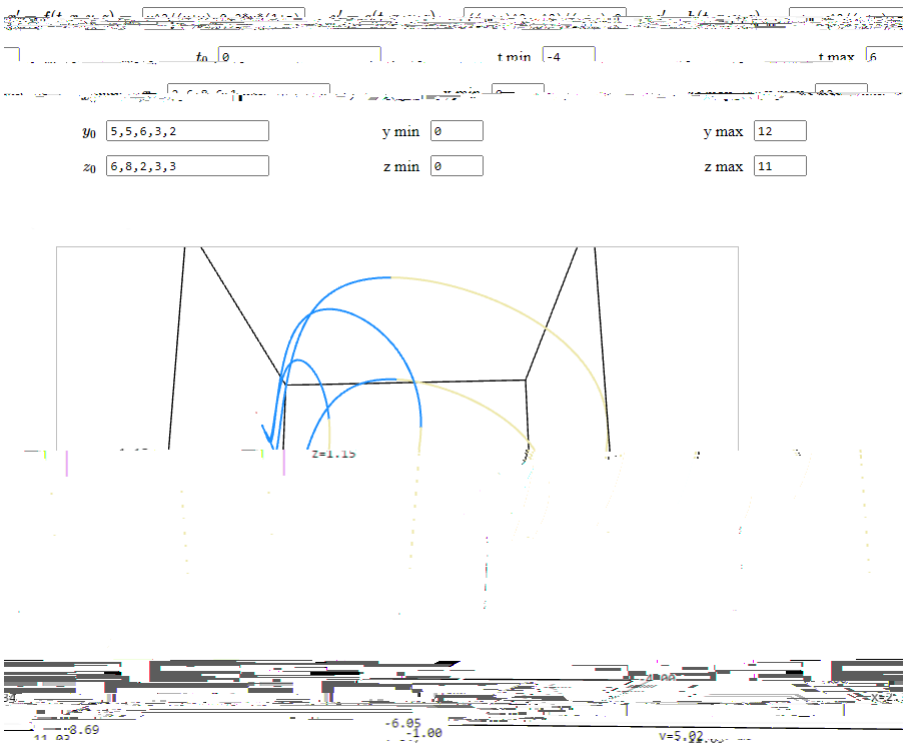

Figure 2: phase portrait of system (3)

The solution of

$$0 = \frac{x^2}{x+y} \quad 0{:}2x(1+z);$$

$$0 = \frac{(x+y)^2 \quad x^2}{x+y} \quad 0{:}5y(1+z);$$

$$0 = (x+y) \quad \frac{z^2}{x+y}{:}$$

defines the equilibrium points of the population system, which are marked in Figure 2 by red points.

Healthcare Technologies and Big Data

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Abstract

An introduction of AI technologies in healthcare determined fundamental changes in healthcare. We are presenting major trends of adopted AI technologies worldwide in healthcare and current progress and challenges of AI adoption in Georgian healthcare systems.

Using technologies in Healthcare System is a part of the global healthcare development strategies. Capturing data and using them for analytic purposes gives possibilities to measure performance of the overall system.

Artificial Intelligence can serve as robust decision support system for healthcare workers, doctors, nurses, healthcare executives, payers, and governing bodies.

AI is based on the data which is processed by healthcare system players. Key players in Data production are doctors and nurses, who have the first access to the information generated during patient care.

AI can simplify the life of all health system workers by performing tasks which must be done by humans in a faster and cost effective way.

Artificial Intelligence is the fastest growing business in the world. According to the CB INSIGHTS 2021 [1] was a year of unprecedented AI private market 0nk (ed)3 (en)of e nowito4 (.)

AI is reinventing modern healthcare through machines that can predict, comprehend, learn, and act.

AI platforms are based on big data sources and during analyzing medical data and predicting or coming up with diagnosis, AI can be the way more trusted that humans, who have limited capabilities, according to the physiological and anatomic specification of the human brain.

AI is using data generated by healthcare players, including doctors, nurses, healthcare managers comeded in HER systems. Also, field scientists, educational and research materials etc.

AI is using Machine Learning (ML) and Deep Learning (DL) Algorithms and statistical models to perform its tasks. Most important in ML and DL is Data captured in the systems using Electronic Health Records EHRs, scientific materials and medical literature.

However, the benefits of AI are very significant, still the field needs to adopt more regulatory rules for maintaining confidentiality of the medical data, raises more ethical concerns and system protection issues. Several countries are in the process of maintaining internal regulations on using AI platforms. E.g., Singapore adopted Artificial Intelligence in Healthcare Guidelines, which consists with all ruled for using AI systems in the country, to minimize the possibilities of the harm on patient care and population health.

Annually more than 400000 inpatients are suffering with preventable harm, with 100000 thousand of death cases, which can be avoided using AI technologies [2].

There are several ways how AI can save lives of patients [3].

1. One of the most common problems during patient care can be delays, which can mean difference between life and death. AI can detect an issue and notify care teams to provide faster treatment decisions and save lives.

2. Using AI can help pathologists to come up with more precise diagnosis.
3. AI algorithms are used to diagnose and treat illness.
4. / [• ‰ o Œ v] v P ‰ o š (} Œ u • Œ µ • š } v o Ç Ì Œ] } o EKGs, genomics, patient medical history and support doctors in decision making process.
5. AI are used for screening purposes to detect cancer at the earliest stage and support diagnostic and treatment process by algorithms.
6. AI is used to identify harmful bacteria in patient blood and laboratory settings.
7. AI is used in Gastroenterology to identify early signs of diseases
8. AI is used in Imaging technologies like Ultrasound, MRI, CT, and X-Ray for diagnostic purposes. Using big data in their applications, systems can come up with more precise diagnosis, compared with doctors.
9. AI is used in research for clinical trials for new treatment tools, drug development etc.
10. Combining AI and physics, AI can support drug development process and it can be done within days, rather than weeks.

Big organizations started to use AI applications during their everyday operations. Here are several examples of how AI helped healthcare organizations perform on a higher quality level with better outcomes in patient care.

Cleveland Clinic in United States started partnership with IBM [4] to build an infrastructure that supports research in areas such as genomics, chemical and drug discovery, and population health.

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AI can be used in healthcare management field, which generates huge amounts of data and usability of these data can be achieved by AI technologies. AI can analyze a data which can be useful for public and population health purposes, for supporting drug

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them to predict healthcare costs and make systems more cost-effective, can support governing regulatory agencies in population health management and planning purposes.

Systems used in Georgian healthcare:

1. Mydoc.ge https://mydoc.ge t already it is used, and they have some statistics.
2. https://www.heaps.ai/index_in.html t

[5] "Johns Hopkins Medicine," 2016. [Online]. Available: https://www.hopkinsmedicine.org/news/media/releases/the_johns_hopkins_hospital_launches_capacity_command_center_to_enhance_hospital_operations.

[6] B. Marr, "Forbes," 2019. [Online]. Available: https://www.forbes.com/sites/bernardmarr/2019/08/16/the-amazing-ways-babylon-health-is-using-artificial-intelligence-to-make-healthcare-universally-accessible/?sh=421f1d713842.